

Integrals



TOPIC 1 Standard Integrals, Integration by Substitution, Integration by Parts



1. $\lim_{x \rightarrow 1} \left(\frac{\int_0^{(x-1)^2} t \cos(t^2) dt}{(x-1) \sin(x-1)} \right)$ [Sep. 06, 2020 (I)]

- (a) is equal to $\frac{1}{2}$ (b) is equal to 1
(c) is equal to $-\frac{1}{2}$ (d) does not exist

2. If $\int (e^{2x} + 2e^x - e^{-x} - 1)e^{(e^x + e^{-x})} dx = g(x)e^{(e^x + e^{-x})} + c$, where c is a constant of integration, then $g(0)$ is equal to: [Sep. 05, 2020 (I)]

- (a) e (b) e^2 (c) 1 (d) 2

3. If $\int \frac{\cos \theta}{5 + 7 \sin \theta - 2 \cos^2 \theta} d\theta = A \log_e |B(\theta)| + C$, where

C is a constant of integration, then $\frac{B(\theta)}{A}$ can be :

[Sep. 05, 2020 (II)]

- (a) $\frac{2 \sin \theta + 1}{\sin \theta + 3}$ (b) $\frac{2 \sin \theta + 1}{5(\sin \theta + 3)}$
(c) $\frac{5(\sin \theta + 3)}{2 \sin \theta + 1}$ (d) $\frac{5(2 \sin \theta + 1)}{\sin \theta + 3}$

4. The integral $\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$ is equal to (where C is a constant of integration) : [Sep. 04, 2020 (I)]

- (a) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$
(b) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$
(c) $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$
(d) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$

5. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$ ($x \geq 0$). Then $f(3) - f(1)$ is equal to : [Sep. 04, 2020 (I)]

- (a) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (b) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$
(c) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (d) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

6. If $\int \sin^{-1} \left(\sqrt{\frac{x}{1+x}} \right) dx = A(x) \tan^{-1}(\sqrt{x}) + B(x) + C$, where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be : [Sep. 03, 2020 (II)]

- (a) $(x+1, -\sqrt{x})$ (b) $(x+1, \sqrt{x})$
(c) $(x-1, -\sqrt{x})$ (d) $(x-1, \sqrt{x})$

7. The integral $\int \frac{dx}{(x+4)^{8/7} (x-3)^{6/7}}$ is equal to: (where C is a constant of integration) [Jan. 9, 2020 (I)]

- (a) $\left(\frac{x-3}{x+4} \right)^{1/7} + C$ (b) $-\left(\frac{x-3}{x+4} \right)^{-1/7} + C$
(c) $\frac{1}{2} \left(\frac{x-3}{x+4} \right)^{3/7} + C$ (d) $-\frac{1}{13} \left(\frac{x-3}{x+4} \right)^{-13/7} + C$

8. If $\int \frac{d\theta}{\cos^2 \theta (\tan 2\theta + \sec 2\theta)} = \lambda \tan \theta + 2 \log_e |f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to: [Jan. 9, 2020 (II)]

- (a) $(1, 1 - \tan \theta)$ (b) $(-1, 1 - \tan \theta)$
(c) $(-1, 1 + \tan \theta)$ (d) $(1, 1 + \tan \theta)$

9. If $\int \frac{\cos x dx}{\sin^3 x (1 + \sin^6 x)^{2/3}} = f(x)(1 + \sin^6 x)^{1/\lambda} + c$ where c is

a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to:

[Jan. 8, 2020 (II)]

- (a) $-\frac{9}{8}$ (b) 2 (c) $\frac{9}{8}$ (d) -2

10. The integral $\int \frac{2x^3-1}{x^4+x} dx$ is equal to :
(Here C is a constant of integration) [April 12, 2019 (I)]
- (a) $\frac{1}{2} \log_e \left| \frac{x^3+1}{x^2} \right| + C$ (b) $\frac{1}{2} \log_e \frac{(x^3+1)^2}{|x^3|} + C$
(c) $\log_e \left| \frac{x^3+1}{x} \right| + C$ (d) $\log_e \left| \frac{x^3+1}{x^2} \right| + C$
11. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = A(x) \cos 2\alpha + B(x) \sin 2\alpha + C$, where C is a constant of integration, then the functions A(x) and B(x) are respectively : [April 12, 2019 (II)]
- (a) $x + \alpha$ and $\log_e |\sin(x + \alpha)|$
(b) $x - \alpha$ and $\log_e |\sin(x - \alpha)|$
(c) $x - \alpha$ and $\log_e |\cos(x - \alpha)|$
(d) $x + \alpha$ and $\log_e |\sin(x - \alpha)|$
12. If $\int \frac{dx}{(x^2 - 2x + 10)^2} = A \left(\tan^{-1} \left(\frac{x-1}{3} \right) + \frac{f(x)}{x^2 - 2x + 10} \right) + C$ where C is a constant of integration, then : [April 10, 2019 (I)]
- (a) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$
(b) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$
(c) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$
(d) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$
13. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set $S = \{x \in \mathbb{R} : f(x) = f(0)\}$ contains exactly: [April 09, 2019 (I)]
- (a) four irrational numbers.
(b) four rational numbers.
(c) two irrational and two rational numbers.
(d) two irrational and one rational number.
14. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to: [April 09, 2019 (I)]
- (a) $-3 \tan^{-1/3} x + C$ (b) $-\frac{3}{4} \tan^{-4/3} x + C$
(c) $-3 \cot^{-1/3} x + C$ (d) $3 \tan^{-1/3} x + C$
(Here C is a constant of integration)
15. If $\int e^{\sec x} (\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)) dx = e^{\sec x} f(x) + C$, then a possible choice of $f(x)$ is: [April 09, 2019 (II)]
- (a) $\sec x + \tan x + \frac{1}{2}$ (b) $\sec x - \tan x - \frac{1}{2}$
(c) $\sec x + x \tan x - \frac{1}{2}$ (d) $x \sec x + \tan x + \frac{1}{2}$
16. $\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx$ is equal to :
(where C is a constant of integration.) [April 08, 2019 (I)]
- (a) $2x + \sin x + 2 \sin 2x + c$ (b) $x + 2 \sin x + 2 \sin 2x + c$
(c) $x + 2 \sin x + \sin 2x + c$ (d) $2x + \sin x + \sin 2x + c$
17. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + C$, where C is a constant of integration, then the function $f(x)$ is equal to : [April 08, 2019 (II)]
- (a) $\frac{3}{x^2}$ (b) $-\frac{1}{6x^3}$ (c) $-\frac{1}{2x^2}$ (d) $-\frac{1}{2x^3}$
18. The integral $\int \cos(\log_e x) dx$ is equal to :
(where C is a constant of integration) [Jan. 12, 2019 (I)]
- (a) $\frac{x}{2} [\sin(\log_e x) - \cos(\log_e x)] + C$
(b) $x [\cos(\log_e x) + \sin(\log_e x)] + C$
(c) $\frac{x}{2} [\cos(\log_e x) + \sin(\log_e x)] + C$
(d) $x [\cos(\log_e x) - \sin(\log_e x)] + C$
19. The integral $\int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx$ is equal to:
(where C is a constant of integration) [Jan. 12, 2019 (II)]
- (a) $\frac{x^4}{6(2x^4 + 3x^2 + 1)^3} + C$ (b) $\frac{x^{12}}{6(2x^4 + 3x^2 + 1)^3} + C$
(c) $\frac{x^4}{(2x^4 + 3x^2 + 1)^3} + C$ (d) $\frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$
20. If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x) \left(\sqrt{1-x^2} \right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals : [Jan. 11, 2019 (I)]
- (a) $\frac{-1}{27x^9}$ (b) $\frac{-1}{3x^3}$ (c) $\frac{1}{27x^6}$ (d) $\frac{1}{9x^4}$

21. If $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x)\sqrt{2x-1} + C$, where C is a constant of integration, then $f(x)$ is equal to: **[Jan. 11, 2019 (II)]**

- (a) $\frac{1}{3}(x+1)$ (b) $\frac{2}{3}(x+2)$
(c) $\frac{2}{3}(x-4)$ (d) $\frac{1}{3}(x+4)$

22. Let $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$

Then $\int \frac{(\sin^n \theta + \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ is equal to:

[Jan 10, 2019(I)]

- (a) $\frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$
(b) $\frac{n}{n^2+1} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$
(c) $\frac{n}{n^2-1} \left(1 + \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$
(d) $\frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n+1} \theta}\right)^{\frac{n+1}{n}} + C$

(where C is a constant of integration)

23. For $x^2 \neq n\pi + 1$, $n \in \mathbb{N}$ (the set of natural numbers), the integral **[Jan. 09, 2019(I)]**

$\int x \sqrt{\frac{2 \sin(x^2-1) - \sin 2(x^2-1)}{2 \sin(x^2-1) + \sin 2(x^2-1)}} dx$ is equal to:

- (a) $\log_e \left| \frac{1}{2} \sec^2(x^2-1) \right| + c$
(b) $\frac{1}{2} \log_e |\sec(x^2-1)| + c$
(c) $\frac{1}{2} \log_e \left| \sec^2 \left(\frac{x^2-1}{2} \right) \right| + c$
(d) $\log_e \left| \sec \left(\frac{x^2-1}{2} \right) \right| + c$

(where c is a constant of integration)

24. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$, ($x \geq 0$),

and $f(0) = 0$, then the value of $f(1)$ is: **[Jan. 09, 2019 (II)]**

- (a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

25. The integral

$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$ is equal to: **[2018]**

- (a) $\frac{-1}{3(1+\tan^3 x)} + C$ (b) $\frac{1}{1+\cot^3 x} + C$
(c) $\frac{-1}{1+\cot^3 x} + C$ (d) $\frac{1}{3(1+\tan^3 x)} + C$

(where C is a constant of integration)

26. If

$$\int \frac{\tan x}{1 + \tan x + \tan^2 x} dx = x - \frac{K}{\sqrt{A}} \tan^{-1} \left(\frac{K \tan x + 1}{\sqrt{A}} \right) + C,$$

(C is a constant of integration), then the ordered pair (K, A) is equal to **[Online April 16, 2018]**

- (a) (2, 3) (b) (2, 1) (c) (-2, 1) (d) (-2, 3)

27. If $f\left(\frac{x-4}{x+2}\right) = 2x+1$, ($x \in \mathbb{R} = \{1, -2\}$), then $\int f(x) dx$ is equal to (where C is a constant of integration) **[Online April 15, 2018]**

- (a) $12 \log_e |1-x| - 3x + c$
(b) $-12 \log_e |1-x| - 3x + c$
(c) $-12 \log_e |1-x| + 3x + c$
(d) $12 \log_e |1-x| + 3x + c$

28. $\int \frac{2x+5}{\sqrt{7-6x-x^2}} dx = A\sqrt{7-6x-x^2} + B \sin^{-1} \left(\frac{x+3}{4} \right) + C$

(where C is a constant of integration), then the ordered pair (A, B) is equal to **[Online April 15, 2018]**

- (a) (-2, -1) (b) (2, -1)
(c) (-2, 1) (d) (2, 1)

29. If $f\left(\frac{3x-4}{3x+4}\right) = x+2$, $x \neq -\frac{4}{3}$, and

$\int f(x) dx = A \log |1-x| + Bx + C$, then the ordered pair (A, B) is equal to: **[Online April 9, 2017]**
(where C is a constant of integration)

- (a) $\left(\frac{8}{3}, \frac{2}{3}\right)$ (b) $\left(-\frac{8}{3}, \frac{2}{3}\right)$
(c) $\left(-\frac{8}{3}, -\frac{2}{3}\right)$ (d) $\left(\frac{8}{3}, -\frac{2}{3}\right)$

30. The integral $\int \sqrt{1+2 \cot x (\operatorname{cosec} x + \cot x)} dx$

$\left(0 < x < \frac{\pi}{2}\right)$ is equal to: **[Online April 8, 2017]**

(where C is a constant of integration)

- (a) $2 \log \left| \sin \frac{x}{2} \right| + C$ (b) $4 \log \left| \sin \frac{x}{2} \right| + C$
(c) $2 \log \left| \cos \frac{x}{2} \right| + C$ (d) $4 \log \left| \cos \frac{x}{2} \right| + C$

31. If $\int \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}} = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then $A + B + C$ equals :
[Online April 9, 2016]
- (a) $\frac{16}{5}$ (b) $\frac{27}{10}$ (c) $\frac{7}{10}$ (d) $\frac{21}{5}$
32. If $\int \frac{\log(t + \sqrt{1+t^2})}{\sqrt{1+t^2}} dt = \frac{1}{2}(g(t))^2 + C$, where C is a constant, then $g(b)$ is equal to :
[Online April 11, 2015]
- (a) $\frac{1}{\sqrt{5}} \log(2 + \sqrt{5})$ (b) $\frac{1}{2} \log(2 + \sqrt{5})$
(c) $2 \log(2 + \sqrt{5})$ (d) $\log(2 + \sqrt{5})$
33. The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x + \frac{1}{x}} dx$ is equal to [2014]
- (a) $(x+1)e^{x + \frac{1}{x}} + c$ (b) $-xe^{x + \frac{1}{x}} + c$
(c) $(x-1)e^{x + \frac{1}{x}} + c$ (d) $xe^{x + \frac{1}{x}} + c$
34. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$ is equal to:
[Online April 12, 2014]
- (a) $\frac{1}{(1 + \cot^3 x)} + c$ (b) $-\frac{1}{3(1 + \tan^3 x)} + c$
(c) $\frac{\sin^3 x}{(1 + \cos^3 x)} + c$ (d) $-\frac{\cos^3 x}{3(1 + \sin^3 x)} + c$
35. The integral $\int x \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx$ ($x > 0$) is equal to:
[Online April 11, 2014]
- (a) $-x + (1+x^2) \tan^{-1} x + c$
(b) $x - (1+x^2) \cot^{-1} x + c$
(c) $-x + (1+x^2) \cot^{-1} x + c$
(d) $x - (1+x^2) \tan^{-1} x + c$
36. $\int \frac{\sin^8 x - \cos^8 x}{(1 - 2 \sin^2 x \cos^2 x)} dx$ is equal to:
[Online April 9, 2014]
- (a) $\frac{1}{2} \sin 2x + c$ (b) $-\frac{1}{2} \sin 2x + c$
(c) $-\frac{1}{2} \sin x + c$ (d) $-\sin^2 x + c$
37. If $\int f(x) dx = \psi(x)$, then $\int x^5 f(x^3) dx$ is equal to [2013]
- (a) $\frac{1}{3} [x^3 \psi(x^3) - \int x^2 \psi(x^3) dx] + C$
(b) $\frac{1}{3} x^3 \psi(x^3) - 3 \int x^3 \psi(x^3) dx + C$
(c) $\frac{1}{3} x^3 \psi(x^3) - \int x^2 \psi(x^3) dx + C$
(d) $\frac{1}{3} [x^3 \psi(x^3) - \int x^3 \psi(x^3) dx] + C$
38. If the integral $\int \frac{\cos 8x + 1}{\cot 2x - \tan 2x} dx = A \cos 8x + k$, where k is an arbitrary constant, then A is equal to :
[Online April 25, 2013]
- (a) $-\frac{1}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$
39. If the $\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$, then a is equal to : [2012]
- (a) -1 (b) -2 (c) 1 (d) 2
40. If $f(x) = \int \left(\frac{x^2 + \sin^2 x}{1 + x^2} \right) \sec^2 x dx$ and $f(0) = 0$, then $f(1)$ equals [Online May 19, 2012]
- (a) $\tan 1 - \frac{\pi}{4}$ (b) $\tan 1 + 1$
(c) $\frac{\pi}{4}$ (d) $1 - \frac{\pi}{4}$
41. The integral of $\frac{x^2 - x}{x^3 - x^2 + x - 1}$ w.r.t. x is [Online May 12, 2012]
- (a) $\frac{1}{2} \log(x^2 + 1) + C$ (b) $\frac{1}{2} \log|x^2 - 1| + C$
(c) $\log(x^2 + 1) + C$ (d) $\log|x^2 - 1| + C$
42. Let $f(x)$ be an indefinite integral of $\cos^3 x$.
Statement 1: $f(x)$ is a periodic function of period π .
Statement 2: $\cos^3 x$ is a periodic function.
[Online May 7, 2012]
- (a) Statement 1 is true, Statement 2 is false.
(b) Both the Statements are true, but Statement 2 is not the correct explanation of Statement 1.
(c) Both the Statements are true, and Statement 2 is correct explanation of Statement 1.
(d) Statement 1 is false, Statement 2 is true.
43. The value of $\sqrt{2} \int \frac{\sin x dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is [2008]

(a) $x + \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + c$

(b) $x - \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c$

(c) $x + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c$

(d) $x - \log \left| \cos \left(x - \frac{\pi}{4} \right) \right| + c$

44. $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$ equals

(a) $\log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$

(b) $\log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + C$

(c) $\frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$

(d) $\frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + C$

45. $\int \frac{dx}{\cos x - \sin x}$ is equal to

(a) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + C$

(b) $\frac{1}{\sqrt{2}} \log \left| \cot \left(\frac{x}{2} \right) \right| + C$

(c) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{3\pi}{8} \right) \right| + C$

(d) $\frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{\pi}{8} \right) \right| + C$

46. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin(x-\alpha) + C$, then value of (A, B) is

(a) $(-\cos \alpha, \sin \alpha)$ (b) $(\cos \alpha, \sin \alpha)$

(c) $(-\sin \alpha, \cos \alpha)$ (d) $(\sin \alpha, \cos \alpha)$

47. $f(x)$ and $g(x)$ are two differentiable functions on $[0, 2]$ such that $f''(x) - g''(x) = 0$, $f'(1) = 2g'(1) = 4$, $f(2) = 3g(2) = 9$ then $f(x) - g(x)$ at $x = 3/2$ is

(a) 0 (b) 2 (c) 10 (d) 5

TOPIC 2

Integration of the Forms: $\int e^x(f(x) + f'(x))dx$, $\int e^{kx}(df(x) + f'(x))dx$,
Integration by Partial Fractions,
Integration of Some Special
Irrational Algebraic Functions,
Integration of Different
Expressions of e^x



[2007]

48. The integral $\int_1^2 e^x \cdot x^x (2 + \log_e x) dx$ equals:

[Sep. 06, 2020 (II)]

(a) $e(4e+1)$

(b) $4e^2 - 1$

(c) $e(4e-1)$

(d) $e(2e-1)$

49. A value of α such that

$\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)} = \log_e \left(\frac{9}{8} \right)$ is : [April 12, 2019 (II)]

(a) -2

(b) $\frac{1}{2}$

(c) $-\frac{1}{2}$

(d) 2

50. If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then $g(-1)$ is equal to : [April 10, 2019 (II)]

(a) -1

(b) 1

(c) $-\frac{5}{2}$

(d) $-\frac{1}{2}$

[2004]

51. If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then $f(x)$ is equal to:

[Jan. 10, 2019 (II)]

(a) $-2x^3 - 1$

(b) $-4x^3 - 1$

(c) $-2x^3 + 1$

(d) $4x^3 + 1$

52. The integral $\int \frac{dx}{(1+\sqrt{x})\sqrt{x-x^2}}$ is equal to :

(where C is a constant of integration)

[Online April 10, 2016]

(a) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}} + C$

(b) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + C$

(c) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + C$

(d) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}} + C$

53. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals :

[2015]

(a) $-(x^4+1)^{\frac{1}{4}} + c$

(b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

(c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

(d) $(x^4+1)^{\frac{1}{4}} + c$

54. The integral $\int \frac{dx}{(x+1)^{\frac{3}{4}}(x-2)^{\frac{5}{4}}}$ is equal to :

[Online April 10, 2015]

- (a) $-\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + C$ (b) $4\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}} + C$
 (c) $4\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}} + C$ (d) $-\frac{4}{3}\left(\frac{x-2}{x+1}\right)^{\frac{1}{4}} + C$

55. If m is a non-zero number and

$$\int \frac{x^{5m-1} + 2x^{4m-1}}{(x^{2m} + x^m + 1)^3} dx = f(x) + c,$$

then $f(x)$ is:

[Online April 19, 2014]

- (a) $\frac{x^{5m}}{2m(x^{2m} + x^m + 1)^2}$ (b) $\frac{x^{4m}}{2m(x^{2m} + x^m + 1)^2}$
 (c) $\frac{2m(x^{5m} + x^{4m})}{(x^{2m} + x^m + 1)^2}$ (d) $\frac{(x^{5m} - x^{4m})}{2m(x^{2m} + x^m + 1)^2}$

56. The integral $\int \frac{x dx}{2-x^2 + \sqrt{2-x^2}}$ equals :

[Online April 23, 2013]

- (a) $\log \left| 1 + \sqrt{2+x^2} \right| + c$ (b) $-\log \left| 1 + \sqrt{2-x^2} \right| + c$
 (c) $-x \log \left| 1 - \sqrt{2-x^2} \right| + c$ (d) $x \log \left| 1 - \sqrt{2+x^2} \right| + c$

57. If $\int \frac{x^2 - x + 1}{x^2 + 1} e^{\cot^{-1} x} dx = A(x) e^{\cot^{-1} x} + C$, then $A(x)$ is equal to :

[Online April 22, 2013]

- (a) $-x$ (b) x (c) $\sqrt{1-x}$ (d) $\sqrt{1+x}$

58. If $\int \frac{dx}{x+x^7} = p(x)$ then, $\int \frac{x^6}{x+x^7} dx$ is equal to:

[Online April 9, 2013]

- (a) $\ln |x| - p(x) + c$ (b) $\ln |x| + p(x) + c$
 (c) $x - p(x) + c$ (d) $x + p(x) + c$

59. $\int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$ is equal to [2005]

- (a) $\frac{\log x}{(\log x)^2 + 1} + C$ (b) $\frac{x}{x^2 + 1} + C$
 (c) $\frac{xe^x}{1+x^2} + C$ (d) $\frac{x}{(\log x)^2 + 1} + C$

TOPIC 3 Evaluation of Definite Integral by Substitution, Properties of Definite Integrals



60. If $I_1 = \int_0^1 (1-x^{50})^{100} dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to : [Sep. 06, 2020 (I)]

- (a) $\frac{5049}{5050}$ (b) $\frac{5050}{5049}$ (c) $\frac{5050}{5051}$ (d) $\frac{5051}{5050}$

61. The value of $\int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} dx$ is: [Sep. 05, 2020 (I)]

- (a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

62. Let $f(x) = |x-2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$.

Then $\int_0^3 (g(x) - f(x)) dx$ is equal to : [Sep. 04, 2020 (I)]

- (a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

63. The integral

$$\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x (2 \sec^2 x \cdot \sin^2 3x + 3 \tan x \cdot \sin 6x) dx$$

is equal to :

[Sep. 04, 2020 (II)]

- (a) $\frac{7}{18}$ (b) $-\frac{1}{9}$ (c) $-\frac{1}{18}$ (d) $\frac{9}{2}$

64. Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\leq x$ respectively of a real number x . If

$\int_0^n \{x\} dx$, $\int_0^n [x] dx$ and $10(n^2 - n)$, ($n \in \mathbb{N}$, $n > 1$) are three consecutive terms of a G.P., then n is equal to [NA Sep. 04, 2020 (II)]

65. $\int_{-\pi}^{\pi} |\pi - |x|| dx$ is equal to : [Sep. 03, 2020 (I)]

- (a) $\sqrt{2}\pi^2$ (b) $2\pi^2$ (c) π^2 (d) $\frac{\pi^2}{2}$

66. If the value of the integral $\int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}} dx$ is $\frac{k}{6}$, then k is equal to : [Sep. 03, 2020 (II)]

- (a) $2\sqrt{3} - \pi$ (b) $2\sqrt{3} + \pi$
 (c) $3\sqrt{2} + \pi$ (d) $3\sqrt{2} - \pi$

67. The integral $\int_0^2 ||x-1| - x| dx$ is equal to _____.

[NA Sep. 02, 2020 (I)]

68. Let $[t]$ denote the greatest integer less than or equal to t . Then the value of $\int_1^2 |2x - [3x]| dx$ is _____.
- [NA Sep. 02, 2020 (II)]
69. If for all real triplets (a, b, c) , $f(x) = a + bx + cx^2$; then $\int_0^1 f(x) dx$ is equal to: [Jan. 9, 2020 (I)]
- (a) $2\left\{3f(1) + 2f\left(\frac{1}{2}\right)\right\}$ (b) $\frac{1}{2}\left\{f(1) + 3f\left(\frac{1}{2}\right)\right\}$
 (c) $\frac{1}{3}\left\{f(0) + f\left(\frac{1}{2}\right)\right\}$ (d) $\frac{1}{6}\left\{f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right\}$
70. The value of $\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$ is equal to: [Jan. 9, 2020 (I)]
- (a) 2π (b) $2\pi^2$ (c) π^2 (d) 4π
71. If $I = \int \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$, then: [Jan. 8, 2020 (II)]
- (a) $\frac{1}{8} < I^2 < \frac{1}{4}$ (b) $\frac{1}{9} < I^2 < \frac{1}{8}$
 (c) $\frac{1}{16} < I^2 < \frac{1}{9}$ (d) $\frac{1}{6} < I^2 < \frac{1}{2}$
72. If $f(a+b+1-x) = f(x)$, for all x , where a and b are fixed positive real numbers, then $\frac{1}{a+b} \int_a^{a+b} x(f(x) + f(x+1)) dx$ is equal to: [Jan. 7, 2020 (I)]
- (a) $\int_{a+1}^{b+1} f(x) dx$ (b) $\int_{a-1}^{b-1} f(x) dx$
 (c) $\int_{a-1}^{b-1} f(x+1) dx$ (d) $\int_{a+1}^{b+1} f(x+1) dx$
73. The value of α for which $4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5$, is: [Jan. 7, 2020 (II)]
- (a) $\log_e 2$ (b) $\log_e \left(\frac{3}{2}\right)$ (c) $\log_e \sqrt{2}$ (d) $\log_e \left(\frac{4}{3}\right)$
74. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2 \theta - \frac{5}{\sin \theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta d\theta$ is equal to: [Jan. 7, 2020 (II)]
- (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{3} + \frac{1}{6}$ (d) $\frac{\pi}{9}$
75. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a continuously differentiable function such that $f(2) = 6$ and $f'(2) = \frac{1}{48}$. If $\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$, then $\lim_{x \rightarrow 2} g(x)$ is equal to: [April 12, 2019 (I)]
- (a) 18 (b) 24 (c) 12 (d) 36
76. If $\int_0^{\frac{x}{2}} \frac{\cot x}{\cot x + \operatorname{cosec} x} dx = m(\pi + n)$, then m, n is equal to: [April 12, 2019 (I)]
- (a) $-\frac{1}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) -1
77. The value of $\int_0^{2\pi} [\sin 2x(1 + \cos 3x)] dx$, where $[t]$ denotes the greatest integer function, is: [April 10, 2019 (I)]
- (a) π (b) $-\pi$ (c) -2π (d) 2π
78. The integral $\int_{\pi/6}^{\pi/3} \sec^{2/3} x \operatorname{cosec}^{4/3} x dx$ is equal to: [April 10, 2019 (II)]
- (a) $3^{5/6} - 3^{2/3}$ (b) $3^{4/3} - 3^{1/3}$
 (c) $3^{7/6} - 3^{5/6}$ (d) $3^{5/3} - 3^{1/3}$
79. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is: [April 9, 2019 (I)]
- (a) $\frac{\pi-2}{8}$ (b) $\frac{\pi-1}{4}$ (c) $\frac{\pi-2}{4}$ (d) $\frac{\pi-1}{2}$
80. The value of the integral $\int_0^1 x \cot^{-1}(1-x^2+x^4) dx$ is: [April 09, 2019 (II)]
- (a) $\frac{\pi}{2} - \frac{1}{2} \log_e 2$ (b) $\frac{\pi}{4} - \log_e 2$
 (c) $\frac{\pi}{2} - \log_e 2$ (d) $\frac{\pi}{4} - \frac{1}{2} \log_e 2$
81. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function and $f(2) = 6$, then $\lim_{x \rightarrow 2} \int_6^{f(x)} \frac{2t dt}{(x-2)}$ is: [April 09, 2019 (II)]
- (a) $24f'(2)$ (b) $2f'(2)$ (c) 0 (d) $12f'(2)$
82. If $f(x) = \frac{2-x \cos x}{2+x \cos x}$ and $g(x) = \log_e x$, ($x > 0$) then the value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} g(f(x)) dx$ is: [April 8, 2019 (I)]
- (a) $\log_e 3$ (b) $\log_e e$ (c) $\log_e 2$ (d) $\log_e 1$

83. Let $f(x) = \int_0^x g(t) dt$, where g is a non-zero even function. If

$f(x+5) = g(x)$, then $\int_0^x f(t) dt$ equals : [April 08, 2019 (II)]

- (a) $\int_{x+5}^5 g(t) dt$ (b) $\int_5^{x+5} g(t) dt$
(c) $2 \int_5^{x+5} g(t) dt$ (d) $5 \int_{x+5}^5 g(t) dt$

84. Let f and g be continuous functions on $[0, a]$ such that

$f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$, then $\int_0^a f(x) g(x) dx$

is equal to: [Jan. 12, 2019 (I)]

- (a) $4 \int_0^a f(x) dx$ (b) $\int_0^a f(x) dx$
(c) $2 \int_0^a f(x) dx$ (d) $-3 \int_0^a f(x) dx$

85. The integral $\int_1^e \left\{ \left(\frac{x}{e} \right)^{2x} - \left(\frac{e}{x} \right)^x \right\} \log_e x dx$ is equal to :

[Jan. 12, 2019 (II)]

- (a) $\frac{1}{2} - e - \frac{1}{e^2}$ (b) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$
(c) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$ (d) $\frac{3}{2} - e - \frac{1}{2e^2}$

86. The value of the integral $\int_{-2}^2 \frac{\sin^2 x}{\left[\frac{x}{\pi} \right] + \frac{1}{2}} dx$

(where $[x]$ denotes the greatest integer less than or equal to x) is : [Jan. 11, 2019 (I)]

- (a) 0 (b) $\sin 4$ (c) 4 (d) $4 - \sin 4$

87. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x (\tan^5 x + \cot^5 x)}$ equals :

[Jan. 11, 2019 (II)]

- (a) $\frac{1}{20} \tan^{-1} \left(\frac{1}{9\sqrt{3}} \right)$ (b) $\frac{1}{10} \left(\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{9\sqrt{3}} \right) \right)$
(c) $\frac{\pi}{40}$ (d) $\frac{1}{5} \left(\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{3\sqrt{3}} \right) \right)$

88. Let $I = \int_a^b (x^4 - 2x^2) dx$. If I is minimum then the ordered pair (a, b) is: [Jan 10, 2019 (I)]

- (a) $(0, \sqrt{2})$ (b) $(-\sqrt{2}, 0)$
(c) $(\sqrt{2}, -\sqrt{2})$ (d) $(-\sqrt{2}, \sqrt{2})$

89. If $\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt$, then $f'(1/2)$ is:

[Jan. 10, 2019 (II)]

- (a) $\frac{24}{25}$ (b) $\frac{18}{25}$ (c) $\frac{4}{5}$ (d) $\frac{6}{25}$

90. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x] + [\sin x] + 4}$, where $[t]$ denotes the greatest integer less than or equal to t , is:

[Jan. 10, 2019 (II)]

- (a) $\frac{1}{12} (7\pi + 5)$ (b) $\frac{1}{12} (7\pi - 5)$
(c) $\frac{3}{20} (4\pi - 3)$ (d) $\frac{3}{10} (4\pi - 3)$

91. The value of $\int_0^{\pi} |\cos x|^3 dx$ is: [Jan 9, 2019 (I)]

- (a) 0 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{4}{3}$

92. Let f be a differentiable function from $\mathbf{R}$ to $\mathbf{R}$ such that $|f'(x) - f'(y)| \leq 2|x - y|^{3/2}$, for all $x, y, \in \mathbf{R}$. If $f(0) = 1$ then

$\int_0^1 f^2(x) dx$ is equal to : [Jan. 09, 2019 (II)]

- (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

93. If $\int_0^{\pi/3} \frac{\tan \theta}{\sqrt{2k \sec \theta}} d\theta = 1 - \frac{1}{\sqrt{2}}$, ($k > 0$) then the value of k is:

[Jan. 09, 2019 (II)]

- (a) 4 (b) $\frac{1}{2}$ (c) 1 (d) 2

94. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$ is : [2018]

- (a) $\frac{\pi}{2}$ (b) 4π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

95. If $f(x) = \int_0^x t(\sin x - \sin t) dt$ then

[Online April 16, 2018]

- (a) $f'''(x) + f'(x) = \cos x - 2x \sin x$
(b) $f'''(x) + f''(x) - f'(x) = \cos x$
(c) $f'''(x) - f''(x) = \cos x - 2x \sin x$
(d) $f'''(x) + f''(x) = \sin x$

96. The value of integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x} dx$ is
[Online April 15, 2018]

(a) $\frac{\pi}{2}(\sqrt{2}+1)$ (b) $\pi(\sqrt{2}-1)$
(c) $2\pi(\sqrt{2}-1)$ (d) $\pi\sqrt{2}$

97. If $I_1 = \int_0^1 e^{-x} \cos^2 x dx$; $I_2 = \int_0^1 e^{-x^2} \cos^2 x dx$ and $I_3 = \int_0^1 e^{-x^3} dx$; then
[Online April 15, 2018]

(a) $I_2 > I_3 > I_1$ (b) $I_3 > I_1 > I_2$
(c) $I_2 > I_1 > I_3$ (d) $I_3 > I_2 > I_1$

98. The value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \left(1 + \log \left(\frac{2+\sin x}{2-\sin x}\right)\right) dx$ is
[Online April 15, 2018]

(a) $\frac{3}{16}\pi$ (b) 0 (c) $\frac{3}{8}\pi$ (d) $\frac{3}{4}$

99. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to : [2017]

(a) -1 (b) -2 (c) 2 (d) 4

100. Let $I_n = \int \tan^n x dx$, ($n > 1$). $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to : [2017]

(a) $\left(-\frac{1}{5}, 0\right)$ (b) $\left(-\frac{1}{5}, 1\right)$ (c) $\left(\frac{1}{5}, 0\right)$ (d) $\left(\frac{1}{5}, -1\right)$

101. If $\int_1^2 \frac{dx}{(x^2-2x+4)^2} = \frac{k}{k+5}$ then k is equal to :
[Online April 9, 2017]

(a) 1 (b) 2 (c) 3 (d) 4

102. The integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \frac{8 \cos 2x}{(\tan x + \cot x)^3} dx$ equals :
[Online April 8, 2017]

(a) $\frac{15}{128}$ (b) $\frac{15}{64}$ (c) $\frac{13}{32}$ (d) $\frac{15}{256}$

103. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$ is equal to : [2016]

(a) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$ (b) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$
(c) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$ (d) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$

104. For $x \in \mathbb{R}$, $x \neq 0$, if $y(x)$ is a differentiable function such

that $\int_1^x y(t) dt = (x+1) \int_1^x ty(t) dt$, then $y(x)$ equals :

(where C is a constant) [Online April 10, 2016]

(a) $Cx^3 e^{\frac{1}{x}}$ (b) $\frac{C}{x^2} e^{-\frac{1}{x}}$ (c) $\frac{C}{x} e^{-\frac{1}{x}}$ (d) $\frac{C}{x^3} e^{-\frac{1}{x}}$

105. The value of the integral

$\int_4^{10} \frac{[x^2] dx}{[x^2 - 28x + 196] + [x^2]}$, where $[x]$

denotes the greatest integer less than or equal to x, is :

[Online April 10, 2016]

(a) $\frac{1}{3}$ (b) 6 (c) 7 (d) 3

106. If $2 \int_0^1 \tan^{-1} x dx = \int_0^1 \cot^{-1}(1-x+x^2) dx$, then

$\int_0^1 \tan^{-1}(1-x+x^2) dx$ is equal to :

[Online April 9, 2016]

(a) $\frac{\pi}{2} + \log 2$ (b) $\log 2$

(c) $\frac{\pi}{2} - \log 4$ (d) $\log 4$

107. The integral [2015]

$\int_2^4 \frac{\log x^2}{2 \log x^2 + \log(36 - 12x + x^2)} dx$ is equal to :

(a) 1 (b) 6 (c) 2 (d) 4

108. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(2-x) = f(2+x)$ and $f(4-x) = f(4+x)$, for all $x \in \mathbb{R}$ and

$\int_0^2 f(x) dx = 5$. Then the value of $\int_{10}^{50} f(x) dx$ is :

[Online April 11, 2015]

(a) 125 (b) 80 (c) 100 (d) 200

109. Let $f: (-1, 1) \rightarrow \mathbb{R}$ be a continuous function. If

$\int_0^{\sin x} f(t) dt = \frac{\sqrt{3}}{2} x$, then $f\left(\frac{\sqrt{3}}{2}\right)$ is equal to :

[Online April 11, 2015]

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{3}$

110. For $x > 0$, let $f(x) = \int_1^x \frac{\log t}{1+t} dt$. Then $f(x) + f\left(\frac{1}{x}\right)$ is equal to:

[Online April 10, 2015]

(a) $\frac{1}{4}(\log x)^2$ (b) $\log x$

(c) $\frac{1}{2}(\log x)^2$ (d) $\frac{1}{4} \log x^2$

111. The integral $\int_0^{\pi} \sqrt{1 + 4 \sin^2 \frac{x}{2}} - 4 \sin \frac{x}{2} dx$ equals: [2014]

- (a) $4\sqrt{3} - 4$ (b) $4\sqrt{3} - 4 - \frac{\pi}{3}$
(c) $\pi - 4$ (d) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

112. Let function F be defined as $F(x) = \int_1^x \frac{e^t}{t} dt$, $x > 0$ then the

value of the integral $\int_1^x \frac{e^t}{t+a} dt$, where $a > 0$, is:

[Online April 19, 2014]

- (a) $e^a [F(x) - F(1+a)]$
(b) $e^{-a} [F(x+a) - F(a)]$
(c) $e^a [F(x+a) - F(1+a)]$
(d) $e^{-a} [F(x+a) - F(1+a)]$

113. If for a continuous function $f(x)$, $\int_{-\pi}^t (f(x) + x) dx = \pi^2 -$

t^2 , for all $t \geq -\pi$, then $f\left(-\frac{\pi}{3}\right)$ is equal to:

[Online April 12, 2014]

- (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

114. If $[]$ denotes the greatest integer function, then the integral $\int_0^{\pi} [\cos x] dx$ is equal to: [Online April 12, 2014]

- (a) $\frac{\pi}{2}$ (b) 0 (c) -1 (d) $-\frac{\pi}{2}$

115. If for $n \geq 1$, $P_n = \int_1^e (\log x)^n dx$, then $P_{10} - 90P_8$ is equal to:

[Online April 11, 2014]

- (a) -9 (b) $10e$ (c) $-9e$ (d) 10

116. The integral $\int_0^{\frac{1}{2}} \frac{\ln(1+2x)}{1+4x^2} dx$, equals:

[Online April 9, 2014]

- (a) $\frac{\pi}{4} \ln 2$ (b) $\frac{\pi}{8} \ln 2$ (c) $\frac{\pi}{16} \ln 2$ (d) $\frac{\pi}{32} \ln 2$

117. The intercepts on x-axis made by tangents to the curve,

$y = \int_0^x |t| dt$, $x \in \mathbb{R}$, which are parallel to the line $y = 2x$, are

equal to :

[2013]

- (a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

118. Statement-1 : The value of the integral

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$$
 is equal to $\pi/6$

Statement-2 : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$. [2013]

- (a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

119. For $0 \leq x \leq \frac{\pi}{2}$, the value of

$$\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t}) dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t}) dt$$
 equals :

[Online April 25, 2013]

- (a) $\frac{\pi}{4}$ (b) 0 (c) 1 (d) $-\frac{\pi}{4}$

120. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx$ is :

[Online April 23, 2013]

- (a) π (b) $\frac{\pi}{2}$ (c) 4π (d) $\frac{\pi}{4}$

121. The integral $\int_{7\pi/4}^{7\pi/3} \sqrt{\tan^2 x} dx$ is equal to :

[Online April 22, 2013]

- (a) $\log 2\sqrt{2}$ (b) $\log 2$
(c) $2 \log 2$ (d) $\log \sqrt{2}$

122. If $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$, then $\frac{d^2 y}{dx^2}$ is equal to :

[Online April 9, 2013]

- (a) y (b) $\sqrt{1+y^2}$
(c) $\frac{x}{\sqrt{1+y^2}}$ (d) y^2

123. If $g(x) = \int_0^x \cos 4t dt$, then $g(x+\pi)$ equals [2012]

- (a) $\frac{g(x)}{g(\pi)}$ (b) $g(x) + g(\pi)$
(c) $g(x) - g(\pi)$ (d) $g(x) \cdot g(\pi)$

124. If $[x]$ is the greatest integer $\leq x$, then the value of the integral

$$\int_{-0.9}^{0.9} \left([x^2] + \log \left(\frac{2-x}{2+x} \right) \right) dx \text{ is } \quad [\text{Online May 26, 2012}]$$

- (a) 0.486 (b) 0.243 (c) 1.8 (d) 0

125. The value of the integral $\int_0^{0.9} [x - 2[x]] dx$,

where $[.]$ denotes the greatest integer function is

[Online May 19, 2012]

- (a) 0.9 (b) 1.8 (c) -0.9 (d) 0

126. If $\frac{d}{dx} G(x) = \frac{e^{\tan x}}{x}$, $x \in (0, \pi/2)$, then

$$\int_{1/4}^{1/2} \frac{2}{x} \cdot e^{\tan(\pi x^2)} dx \text{ is equal to } \quad [\text{Online May 12, 2012}]$$

- (a) $G(\pi/4) - G(\pi/16)$ (b) $2[G(\pi/4) - G(\pi/16)]$

- (c) $\pi[G(1/2) - G(1/4)]$ (d) $G(1/\sqrt{2}) - G(1/2)$

127. If $\int_e^x t f(t) dt = \sin x - x \cos x - \frac{x^2}{2}$, for all $x \in \mathbb{R} - \{0\}$,

then the value of $f\left(\frac{\pi}{6}\right)$ is [Online May 7, 2012]

- (a) 1/2 (b) 1 (c) 0 (d) -1/2

128. Let $[.]$ denote the greatest integer function then the value

$$\text{of } \int_0^{1.5} x[x^2] dx \text{ is : } \quad [2011 \text{ RS}]$$

- (a) 0 (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$

129. The value of $\int_0^1 \frac{8 \log(1+x)}{1+x^2} dx$ is [2011]

- (a) $\frac{\pi}{8} \log 2$ (b) $\frac{\pi}{2} \log 2$ (c) $\log 2$ (d) $\pi \log 2$

130. Let $p(x)$ be a function defined on $\mathbb{R}$ such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$.

Then $\int_0^1 p(x) dx$ equals [2010]

- (a) 21 (b) 41 (c) 42 (d) $\sqrt{41}$

131. $\int_0^\pi [\cot x] dx$, where $[.]$ denotes the greatest integer function, is equal to : [2009]

- (a) 1 (b) -1 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

132. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx$. Then which one of the following is true? [2008]

- (a) $I > \frac{2}{3}$ and $J > 2$ (b) $I < \frac{2}{3}$ and $J < 2$

- (c) $I < \frac{2}{3}$ and $J > 2$ (d) $I > \frac{2}{3}$ and $J < 2$

133. The solution for x of the equation

$$\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2} \text{ is } \quad [2007]$$

- (a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$
(c) 2 (d) None of these

134. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_l^x \frac{\log t}{1+t} dt$. Then

$F(e)$ equals [2007]

- (a) 1 (b) 2 (c) 1/2 (d) 0

135. The value of $\int_1^a [x] f'(x) dx$, $a > 1$ where $[x]$ denotes the greatest integer not exceeding x is [2006]

- (a) $af(a) - \{f(1) + f(2) + \dots + f([a])\}$
(b) $[a]f(a) - \{f(1) + f(2) + \dots + f([a])\}$
(c) $[a]f([a]) - \{f(1) + f(2) + \dots + f(a)\}$
(d) $af([a]) - \{f(1) + f(2) + \dots + f(a)\}$

136. $\int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} [(x+\pi)^3 + \cos^2(x+3\pi)] dx$ is equal to [2006]

- (a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32} + \frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4} - 1$

137. $\int_0^\pi x f(\sin x) dx$ is equal to [2006]

- (a) $\pi \int_0^\pi f(\cos x) dx$ (b) $\pi \int_0^\pi f(\sin x) dx$

- (c) $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x) dx$ (d) $\pi \int_0^{\pi/2} f(\cos x) dx$

138. The value of integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$ is [2006]

- (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) 1

139. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, $a > 0$, is [2005]

- (a) $a\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{a}$ (d) 2π

140. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then [2005]

- (a) $I_2 > I_1$ (b) $I_1 > I_2$ (c) $I_3 = I_4$ (d) $I_3 > I_4$

141. Let $f: R \rightarrow R$ be a differentiable function having $f(2) = 6$,

$f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x \rightarrow 2} \int_6^{f(x)} \frac{4t^3}{x-2} dt$ equals [2005]

- (a) 24 (b) 36 (c) 12 (d) 18

142. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$

and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$,

then the value of $\frac{I_2}{I_1}$ is [2004]

- (a) 1 (b) -3 (c) -1 (d) 2

143. If $\int_0^{\pi} xf(\sin x)dx = A \int_0^{\pi/2} f(\sin x)dx$, then A is [2004]

- (a) 2π (b) π (c) $\frac{\pi}{4}$ (d) 0

144. The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is [2004]

- (a) 3 (b) 1 (c) 2 (d) 0

145. The value of $\int_{-2}^3 |1-x^2| dx$ is [2004]

- (a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$

146. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is [2003]

- (a) $\frac{1}{n+1} + \frac{1}{n+2}$ (b) $\frac{1}{n+1}$
(c) $\frac{1}{n+2}$ (d) $\frac{1}{n+1} - \frac{1}{n+2}$

147. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0)=1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral

$\int_0^1 f(x)g(x)dx$, is [2003]

- (a) $e + \frac{e^2}{2} + \frac{5}{2}$ (b) $e - \frac{e^2}{2} - \frac{5}{2}$
(c) $e + \frac{e^2}{2} - \frac{3}{2}$ (d) $e - \frac{e^2}{2} - \frac{3}{2}$

148. If $f(a+b-x) = f(x)$ then $\int_a^b xf(x)dx$ is equal to [2003]

- (a) $\frac{a+b}{2} \int_a^b f(a+b+x)dx$ (b) $\frac{a+b}{2} \int_a^b f(b-x)dx$
(c) $\frac{a+b}{2} \int_a^b f(x)dx$ (d) $\frac{b-a}{2} \int_a^b f(x)dx$

149. The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$ is [2003]

- (a) 0 (b) 3 (c) 2 (d) 1

150. If $f(y) = e^y$, $g(y) = y$; $y > 0$ and

$F(t) = \int_0^t f(t-y)g(y)dy$, then [2003]

- (a) $F(t) = te^{-t}$ (b) $F(t) = 1 - te^{-t}(1+t)$
(c) $F(t) = e^t - (1+t)$ (d) $F(t) = te^t$

151. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is [2002]

- (a) $\frac{\pi^2}{4}$ (b) π^2 (c) zero (d) $\frac{\pi}{2}$

152. $\int_0^2 [x^2] dx$ is [2002]

- (a) $2 - \sqrt{2}$ (b) $2 + \sqrt{2}$
(c) $\sqrt{2} - 1$ (d) $-\sqrt{2} - \sqrt{3} + 5$

153. $I_n = \int_0^{\pi/4} \tan^n x dx$ then $\lim_{n \rightarrow \infty} n[I_n + I_{n+2}]$ equals [2002]

- (a) $\frac{1}{2}$ (b) 1 (c) ∞ (d) zero

154. $\int_0^{10\pi} |\sin x| dx$ is [2002]

- (a) 20 (b) 8 (c) 10 (d) 18

TOPIC 4

**Reduction Formulae for Definite
Integration, Gamma & Beta
Function, Walli's Formula,
Summation of Series by
Integration**


155. Let a function $f: [0, 5] \rightarrow \mathbf{R}$ be continuous, $f(1) = 3$ and F be defined as:

$$F(x) = \int_1^x t^2 g(t) dt, \text{ where } g(t) = \int_1^x f(u) du$$

Then for the function F , the point $x = 1$ is :

[Jan. 9, 2020 (II)]

- (a) a point of local minima.
(b) not a critical point.
(c) a point of local maxima.
(d) a point of inflection.

156. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)^{1/3}}{n^{4/3}} + \frac{(n+2)^{1/3}}{n^{4/3}} + \dots + \frac{(2n)^{1/3}}{n^{4/3}} \right)$ is equal to :

[April 10, 2019 (I)]

- (a) $\frac{3}{4}(2)^{4/3} - \frac{3}{4}$ (b) $\frac{4}{3}(2)^{4/3}$
(c) $\frac{3}{2}(2)^{4/3} - \frac{4}{3}$ (d) $\frac{4}{3}(2)^{3/4}$

157. $\lim_{n \rightarrow \infty} \left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \dots + \frac{1}{5n} \right)$ is equal to :

[Jan. 12, 2019 (II)]

- (a) $\frac{\pi}{4}$ (b) $\tan^{-1}(3)$ (c) $\frac{\pi}{2}$ (d) $\tan^{-1}(2)$

158. If $\lim_{n \rightarrow \infty} \frac{1^a + 2^a + \dots + n^a}{(n+1)^{a-1} [(na+2) + \dots + (na+n)]} = \frac{1}{60}$

for some positive real number a , then a is equal to :

[Online April 9, 2017]

- (a) 7 (b) 8 (c) $\frac{15}{2}$ (d) $\frac{17}{2}$

159. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)(n+2)\dots 3n}{n^{2n}} \right)^{\frac{1}{n}}$ is equal to: [2016]

- (a) $\frac{9}{e^2}$ (b) $3 \log 3 - 2$
(c) $\frac{18}{e^4}$ (d) $\frac{27}{e^2}$

160. $f(x) = \int \frac{dx}{\sin^6 x}$ is a polynomial of degree

[Online May 26, 2012]

- (a) 5 in $\cot x$ (b) 5 in $\tan x$
(c) 3 in $\tan x$ (d) 3 in $\cot x$

161. $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{2}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right]$ equals

[2005]

- (a) $\frac{1}{2} \sec 1$ (b) $\frac{1}{2} \operatorname{cosec} 1$
(c) $\tan 1$ (d) $\frac{1}{2} \tan 1$

162. $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{\frac{r}{n}}$ is

[2004]

- (a) $e+1$ (b) $e-1$ (c) $1-e$ (d) e

163. $\lim_{n \rightarrow \infty} \frac{1+2^4+3^4+\dots+n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1+2^3+3^3+\dots+n^3}{n^5}$ [2003]

- (a) $\frac{1}{5}$ (b) $\frac{1}{30}$ (c) Zero (d) $\frac{1}{4}$

164. $\lim_{n \rightarrow \infty} \frac{1^p + 2^p + 3^p + \dots + n^p}{n^{p+1}}$ is

[2002]

- (a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$
(c) $\frac{1}{p} - \frac{1}{p-1}$ (d) $\frac{1}{p+2}$



Hints & Solutions



1. (Bonus)

$$\lim_{x \rightarrow 1} \frac{\frac{1}{2} \sin(x-1)^4}{(x-1) \sin(x-1)}$$

Let $x-1 = h$ when $x \rightarrow 1$ then $h \rightarrow 0$

$$\lim_{h \rightarrow 0} \frac{\sin h^4}{h^4} \times \frac{h}{\sin h} \times h^2 = 1 \times 1 \times 0 = 0$$

(No any option is correct)

2. (d) $\int (e^{2x} + 2e^x - e^{-x} - 1) \cdot e^{(e^x + e^{-x})} dx$

$$I = \int (e^{2x} + e^x - 1) \cdot e^{(e^x + e^{-x})} dx + \int (e^x - e^{-x}) e^{(e^x + e^{-x})} dx$$

$$= \int e^x (e^x + 1 - e^{-x}) \cdot e^{(e^x + e^{-x})} dx + e^{(e^x + e^{-x})}$$

$$= \int (e^x - e^{-x} + 1) e^{(e^x + e^{-x} + x)} dx + e^{(e^x + e^{-x})}$$

Let $e^x + e^{-x} + x = t \Rightarrow (e^x + e^{-x} + 1) dx = dt$

$$= \int e^t dt + e^{(e^x + e^{-x})} = e^t + e^{(e^x + e^{-x})} + C$$

$$= e^{(e^x + e^{-x} + x)} + e^{(e^x + e^{-x})} + C$$

$$= (e^x + 1) \cdot e^{(e^x + e^{-x})} + C$$

So, $g(x) = 1 + e^x$ and $g(0) = 2$

3. (d) Let $\sin \theta = t \Rightarrow \cos \theta d\theta = dt$

$$\int \frac{\cos \theta}{5 + 7 \sin \theta - 2 \cos^2 \theta} d\theta = \frac{dt}{5 + 7t - 2 + 2t^2}$$

$$\Rightarrow \frac{1}{2} \int \frac{dt}{\left(t + \frac{7}{4}\right)^2 - \left(\frac{5}{4}\right)^2} = \frac{1}{5} \ln \left| \frac{t + \frac{1}{2}}{t + 3} \right| + C$$

$$= \frac{1}{5} \ln \left| \frac{2t + 1}{t + 3} \right| + C = \frac{1}{5} \ln \left| \frac{2 \sin \theta + 1}{\sin \theta + 3} \right| + C$$

$$\therefore B(\theta) = \frac{2 \sin \theta + 1}{2(\sin \theta + 3)} \text{ and } A = \frac{1}{5}$$

$$\Rightarrow \frac{B(\theta)}{A} = \frac{5(2 \sin \theta + 1)}{(\sin \theta + 3)}$$

4. (a) $\int \frac{x^2}{(x \sin x + \cos x)^2} dx$

$$\therefore \frac{d}{dx} (x \sin x + \cos x) = x \cos x$$

$$= \int \frac{x \cos x}{(x \sin x + \cos x)^2} \left(\frac{x}{\cos x} \right) dx$$

$$= \frac{x}{\cos x} \left[\frac{-1}{x \sin x + \cos x} \right]$$

$$- \int \frac{x \sin x + \cos x}{\cos^2 x} \left[\frac{-1}{x \sin x + \cos x} \right] dx$$

$$= \frac{x}{\cos x} \left[\frac{-1}{x \sin x + \cos x} \right] + \int \sec^2 x dx$$

$$= \frac{-x \sec x}{x \sin x + \cos x} + \tan x + C$$

5. (d) $\int \frac{\sqrt{x}}{(1+x)^2} dx \quad (x > 0)$

Put $x = \tan^2 \theta \Rightarrow 2x dx = 2 \tan \theta \sec^2 \theta d\theta$

$$I = \int \frac{2 \tan^2 \theta \cdot \sec^2 \theta}{\sec^4 \theta} d\theta = \int 2 \sin^2 \theta d\theta$$

$$= \theta - \frac{\sin 2\theta}{2} + C$$

$$\Rightarrow f(x) = \theta - \frac{1}{2} \times \frac{2 \tan \theta}{1 + \tan^2 \theta} + C$$

$$f(x) = \theta - \frac{\tan \theta}{1 + \tan^2 \theta} + C = \tan^{-1} \sqrt{x} - \frac{\sqrt{x}}{1+x} + C$$

$$\text{Now } f(3) - f(1) = \tan^{-1}(\sqrt{3}) - \frac{\sqrt{3}}{1+3} - \tan^{-1}(1) + \frac{1}{2}$$

$$= \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

6. (a) $I = \int \sin^{-1} \left(\frac{\sqrt{x}}{\sqrt{1+x}} \right) dx = \int \tan^{-1} \sqrt{x} \cdot 1 dx$

I II

$$= x \tan^{-1} \sqrt{x} - \int \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} x dx + C$$

$$= x \tan^{-1} \sqrt{x} - \frac{1}{2} \int \frac{t \cdot 2t dt}{1+t^2} + C$$

$$(\text{Put } x = t^2 \Rightarrow dx = 2t dt)$$

$$= x \tan^{-1} \sqrt{x} - \int \frac{t^2}{1+t^2} dt + C$$

$$= x \tan^{-1} \sqrt{x} - t + \tan^{-1} t + C$$

$$= x \tan^{-1} \sqrt{x} - \sqrt{x} + \tan^{-1} \sqrt{x} + C$$

$$= (x+1) \tan^{-1} \sqrt{x} - \sqrt{x} + C$$

$$\Rightarrow A(x) = x+1 \Rightarrow B(x) = -\sqrt{x}$$

$$7. \quad (a) \quad I = \int \frac{dx}{(x+4)^{8/7} (x-3)^{6/7}}$$

$$= \int \left(\frac{x-3}{x+4} \right)^{-6/7} \frac{1}{(x+4)^2} dx$$

$$\text{Let } \frac{x-3}{x+4} = t^7,$$

Differentiate on both sides, we get

$$\frac{7}{(x+4)^2} dx = 7t^6 dt$$

$$\text{Hence, } I = \int t^{-6} t^6 dt = t + C = \left(\frac{x-3}{x+4} \right)^{1/7} + C$$

$$8. \quad (c) \quad I = \int \frac{d\theta}{\cos^2 \theta (\tan 2\theta + \sec 2\theta)}$$

$$= \int \frac{\sec^2 \theta}{\frac{1+\tan^2 \theta}{1-\tan^2 \theta} + \frac{2 \tan \theta}{1-\tan^2 \theta}} d\theta$$

$$= \int \frac{\sec^2 \theta (1-\tan^2 \theta)}{(1+\tan \theta)^2} d\theta$$

$$= \int \frac{\sec^2 \theta (1-\tan \theta)}{1+\tan \theta} d\theta$$

Let $\tan \theta = t \Rightarrow \sec^2 \theta d\theta = dt$, then

$$I = \int \left(\frac{1-t}{1+t} \right) dt = \int \left(-1 + \frac{2}{1+t} \right) dt$$

$$= -t + 2 \log(1+t) + C$$

$$= -\tan \theta + 2 \log(1+\tan \theta) + C$$

Hence, by comparison $\lambda = -1$ and $f(x) = 1 + \tan \theta$

$$9. \quad (d) \quad \text{Let } I = \int \frac{\cos x dx}{\sin^3 x (1 + \sin^6 x)^{2/3}}$$

$$= f(x) (1 + \sin^6 x)^{1/3} + c$$

...(i)

$$\text{If } \sin x = t$$

$$\text{then, } \cos x dx = dt$$

$$I = \int \frac{dt}{t^3 \left(1 + t^6 \right)^{2/3}} = \int \frac{dt}{t^7 \left(1 + \frac{1}{t^6} \right)^{2/3}}$$

$$\text{Put } 1 + \frac{1}{t^6} = r^3 \Rightarrow \frac{dt}{t^7} = \frac{-1}{2} r^2 dr$$

$$= -\frac{1}{2} \int \frac{r^2 dr}{r^2} = -\frac{1}{2} r + c$$

$$= -\frac{1}{2} \left(\frac{\sin^6 x + 1}{\sin^6 x} \right)^{1/3} + c = -\frac{1}{2 \sin^2 x} (1 + \sin^6 x)^{1/3} + c$$

$$f(x) = -\frac{1}{2} \operatorname{cosec}^2 x \text{ and } \lambda = 3 \quad [\text{from eqn. (i)}]$$

$$\therefore \lambda f\left(\frac{\pi}{3}\right) = -2$$

$$10. \quad (c) \quad \text{Given integral, } I = \int \frac{(2x^3 - 1) dx}{x^4 + x} = \int \frac{(2x - x^{-2}) dx}{x^2 + x^{-1}}$$

$$\text{Put } x^2 + x^{-1} = u \Rightarrow (2x - x^{-2}) dx = du$$

$$\Rightarrow I = \int \frac{du}{u} = \log |u| + c = \log |x^2 + x^{-1}| + c$$

$$= \log \left| \frac{x^3 + 1}{x} \right| + c$$

$$11. \quad (b) \quad \text{Given integral}$$

$$\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = \int \frac{\sin(x + \alpha)}{\sin(x - \alpha)} dx$$

$$\text{Let } x - \alpha = t \Rightarrow dx = dt$$

$$= \int \frac{\sin(t + 2\alpha)}{\sin t} dt = \int [\cos 2\alpha + \sin 2\alpha \cdot \cot t] dt$$

$$= \cos 2\alpha \cdot t + \sin 2\alpha \cdot \log |\sin t| + c$$

$$= (x - \alpha) \cdot \cos 2\alpha + \sin 2\alpha \cdot \log |\sin(x - \alpha)| + c$$

$$12. \quad (a) \quad \text{Let } I = \int \frac{dx}{(x^2 - 2x + 10)^2} = \int \frac{dx}{((x-1)^2 + 9)^2}$$

$$\text{Let } (x-1)^2 = 9 \tan^2 \theta$$

...(i)

$$\Rightarrow \tan \theta = \frac{x-1}{3}$$

After differentiating equation ... (i), we get

$$2(x-1)dx = 18 \tan \theta \sec^2 \theta d\theta$$

$$\therefore I = \int \frac{18 \tan \theta \sec^2 \theta d\theta}{2 \times 3 \tan \theta \times 81 \sec^4 \theta}$$

$$I = \frac{1}{27} \int \cos^2 \theta d\theta = \frac{1}{27} \times \frac{1}{2} \int (1 + \cos 2\theta) d\theta$$

$$I = \frac{1}{54} \left\{ \theta + \frac{\sin 2\theta}{2} \right\} + c$$

$$I = \frac{1}{54} \left[\tan^{-1} \left(\frac{x-1}{3} \right) + \frac{1}{2} \times \frac{2 \left(\frac{x-1}{3} \right)}{1 + \left(\frac{x-1}{3} \right)^2} \right] + c$$

$$I = \frac{1}{54} \left[\tan^{-1} \left(\frac{x-1}{3} \right) + \frac{3(x-1)}{x^2 - 2x + 10} \right] + c$$

Compare it with $A \left[\tan^{-1} \left(\frac{x-1}{b} \right) + \frac{f(x)}{x^2 - 2x + 10} \right] + c$,

we get: $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

- 13. (d)** Since, function $f(x)$ have local extreem points at $x = -1, 0, 1$. Then

$$f(x) = K(x+1)x(x-1)$$

$$= K(x^3 - x)$$

$$\Rightarrow f(x) = K \left(\frac{x^4}{4} - \frac{x^2}{2} \right) + C \quad (\text{using integration})$$

$$\Rightarrow f(0) = C$$

$$\therefore f(x) = f(0) \Rightarrow K \left(\frac{x^4}{4} - \frac{x^2}{2} \right) = 0$$

$$\Rightarrow \frac{x^2}{2} \left(\frac{x^2}{2} - 1 \right) = 0 \Rightarrow x = 0, 0, \sqrt{2}, -\sqrt{2}$$

$$\therefore S = \{0, -\sqrt{2}, \sqrt{2}\}$$

- 14. (a)** $I = \int \sec^3 x \cdot \csc^3 x dx$

$$I = \int \frac{\sec^2 x dx}{\tan^3 x}$$

Put $\tan x = z$

$$\Rightarrow \sec^2 x dx = dz$$

$$\Rightarrow I = \int z^{-\frac{4}{3}} \cdot dz = \frac{z^{-\frac{1}{3}}}{-\frac{1}{3}} + C \Rightarrow I = -3(\tan x)^{-\frac{1}{3}} + C$$

- 15. (a)** Given,

$$\int e^{\sec x} \left(\sec x \tan x f(x) + (\sec x + \tan x + \sec^2 x) \right) dx = e^{\sec x} f(x) + C \quad \dots (i)$$

$$\therefore \int e^{g(x)} \left((g'(x)f(x)) + f'(x) \right) dx = e^{g(x)} f(x) + C$$

Our comparing above equation by equation (i),

$$f(x) = \int (\sec x \tan x + \sec^2 x) dx$$

$$\therefore f(x) = \sec x + \tan x + C$$

16. (c) $\int \frac{\sin \left(\frac{5x}{2} \right)}{\sin \left(\frac{x}{2} \right)} dx = \int \frac{2 \cos \frac{x}{2} \cdot \sin \frac{5x}{2}}{2 \cos \frac{x}{2} \cdot \sin \frac{x}{2}} dx$

$$= \int \frac{\sin 3x + \sin 2x}{\sin x} dx$$

$$= \int (3 - 4 \sin^2 x + 2 \cos x) dx$$

$$[\because \sin 2x = 2 \sin x \cos x \text{ and } \sin 3x = 3 \sin x - 4 \sin^3 x]$$

$$= \int (3 - 2(1 - \cos 2x) + 2 \cos x) dx$$

$$= \int (1 + 2 \cos x + 2 \cos 2x) dx$$

$$= x + 2 \sin x + \sin 2x + c$$

17. (d) Let, $\int \frac{dx}{x^3(1+x^6)^{\frac{2}{3}}} = \int \frac{dx}{x^7(1+x^{-6})^{\frac{2}{3}}}$

Put $1 + x^{-6} = t^3 \Rightarrow -6^{-7} dx = 3t^2 dt \Rightarrow \frac{dx}{x^7} = \left(-\frac{1}{2} \right) t^2 dt$

Now, $I = \int \left(-\frac{1}{2} \right) \frac{t^2 dt}{t^2} = -\frac{1}{2} t + C$

$$= -\frac{1}{2} (1 + x^{-6})^{\frac{1}{3}} + C = -\frac{1}{2} \frac{(1 + x^6)^{\frac{1}{3}}}{x^2} + C$$

$$= -\frac{1}{2x^3} x(1 + x^6)^{\frac{1}{3}} + C$$

Hence, $f(x) = -\frac{1}{2x^3}$

18. (c) Let the integral, $I = \int \cos(\ln x) dx$

$$\begin{aligned}\Rightarrow I &= \cos(\ln x) \cdot x - \int \frac{-\sin(\ln x)}{x} \cdot x dx \\ &= x \cos(\ln x) + \int \sin(\ln x) dx \\ &= x \cos(\ln x) + \sin(\ln x) \cdot x - \int \frac{\cos(\ln x)}{x} \cdot x dx \\ &= x \cos(\ln x) + \sin(\ln x) \cdot x - I \\ \Rightarrow 2I &= x(\cos(\ln x) + \sin(\ln x)) + C \\ \Rightarrow I &= \frac{x}{2} [\cos(\ln x) + \sin(\ln x)] + C\end{aligned}$$

19. (b) $I = \int \frac{3x^{13} + 2x^{11}}{(2x^4 + 3x^2 + 1)^4} dx = \int \frac{3x^{13} + 2x^{11}}{x^{16} \left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right)^4} dx$

$$I = \int \frac{\frac{3}{x^3} + \frac{2}{x^5}}{\left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right)^4} dx$$

Let $2 + \frac{3}{x^2} + \frac{1}{x^4} = t$, $-2\left(\frac{3}{x^3} + \frac{2}{x^5}\right) dx = dt$

Then, $I = \int \frac{-\frac{dt}{2}}{t^4} = -\frac{1}{2} \frac{t^{-4+1}}{-4+1} + C$

$$I = \frac{-1}{2} \times \frac{1}{(-3)} \frac{1}{\left(2 + \frac{3}{x^2} + \frac{1}{x^4}\right)^3} + C$$

$$I = \frac{1}{6} \frac{x^{12}}{(2x^4 + 3x^2 + 1)^3} + C$$

20. (a) $A(x) \left(\sqrt{1-x^2}\right)^m + C = \int \frac{\sqrt{1-x^2}}{x^4} dx$

$$= \int \frac{\sqrt{\frac{1}{x^2} - 1}}{x^3} dx$$

Let $\frac{1}{x^2} - 1 = u^2$

$$\Rightarrow -\frac{2}{x^3} = \frac{2u du}{dx}$$

$$\frac{dx}{x^3} = -u du$$

$$A(x) \left(\sqrt{1-x^2}\right)^m + C = \int (-u^2) du = -\frac{u^3}{3} + C$$

$$\begin{aligned}&= -\frac{1}{3} \left(\frac{1}{x^2} - 1\right)^{\frac{3}{2}} + C \\ &= -\frac{1}{3} \cdot \frac{1}{x^3} \cdot (1-x^2)^{\frac{3}{2}} + C \\ &= \frac{-1}{3x^3} \left(\sqrt{1-x^2}\right)^3 + C\end{aligned}$$

Compare both sides,

$$\Rightarrow A(x) = -\frac{1}{3x^3} \text{ and } m = 3$$

$$\Rightarrow (A(x))^3 = \frac{-1}{27x^9}$$

21. (c) Let $I = \int \frac{x+1}{\sqrt{2x-1}} dx$

Put $\sqrt{2x-1} = t$

$$\therefore 2x-1 = t^2 \Rightarrow dx = t dt$$

$$I = \int \frac{(t^2+3)}{2} dt = \frac{t^3}{6} + \frac{3t}{2} + C$$

$$= \frac{(2x-1)^{\frac{3}{2}}}{6} + \frac{3}{2} (2x-1)^{\frac{1}{2}} + C$$

$$= \sqrt{2x-1} \left(\frac{x+4}{3}\right) + C$$

$$= f(x) \cdot \sqrt{2x-1} + C$$

Hence, $f(x) = \frac{x+4}{3}$

22. (a) Let, $I = \int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$

Let $\sin \theta = u$

$$\Rightarrow \cos \theta d\theta = du$$

$$\therefore I = \int \frac{(u^n - u)^{\frac{1}{n}}}{u^{n+1}} du$$

$$= \int \frac{\left(1 - \frac{1}{u^{n-1}}\right)^{\frac{1}{n}}}{u^n} du = \int u^{-n} (1 - u^{1-n})^{\frac{1}{n}} du$$

Let $1 - u^{1-n} = v$

$$\Rightarrow -(1-n)u^{-n} du = dv$$

$$\Rightarrow u^{-n} du = \frac{dv}{n-1}$$

$$\begin{aligned}\therefore I &= \int v^n \cdot \frac{dv}{n-1} = \frac{1}{n-1} \cdot \frac{v^{n+1}}{n+1} \\ &= \frac{n}{n^2-1} v^{\frac{n+1}{n}} + C = \frac{n}{n^2-1} \left(1 - \frac{1}{u^{n-1}} \right)^{\frac{n+1}{n}} + C \\ &= \frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n-1} \theta} \right)^{\frac{n+1}{n}} + C\end{aligned}$$

23. (c, d) Consider the given integral

$$I = \int x \sqrt{\frac{2\sin(x^2-1) - 2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1) + 2\sin(x^2-1)\cos(x^2-1)}} dx$$

($\therefore \sin 2\theta = 2\sin \theta \cos \theta$)

$$\Rightarrow I = \int x \sqrt{\frac{1 - \cos(x^2-1)}{1 + \cos(x^2-1)}} dx$$

$$\Rightarrow I = \int x \left| \tan \left(\frac{x^2-1}{2} \right) \right| dx,$$

$$\text{Now let } \frac{x^2-1}{2} = t \Rightarrow \frac{2x}{2} dx = dt$$

$$\therefore I = \int |\tan(t)| dt = \ln |\sec t| + C$$

$$\text{or } I = \ln \left| \sec \left(\frac{x^2-1}{2} \right) \right| + c = \frac{1}{2} \ln \left| \sec^2 \left(\frac{x^2-1}{2} \right) \right| + c$$

24. (a) $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx, x \geq 0$

$$= \int \frac{5x^8 + 7x^6}{x^{14}(x^{-5} + x^{-7} + 2)^2} dx$$

$$= \int \frac{5x^{-6} + 7x^{-8}}{(2 + x^{-7} + x^{-5})^2} dx$$

$$\text{Let } 2 + x^{-7} + x^{-5} = t$$

$$\Rightarrow (-7x^{-8} - 5x^{-6})dx = dt$$

$$\Rightarrow f(x) = \int \frac{-dt}{t^2} = \int -t^{-2} dt = t^{-1} + c$$

$$\Rightarrow f(x) = \frac{1}{2 + x^{-7} + x^{-5}} + c, f(0) = 0 \Rightarrow c = 0$$

$$\therefore f(1) = \frac{1}{4}$$

25. (a) Let I

$$= \int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

$$= \int \frac{\sin^2 x \cos^2 x}{[(\sin^2 x + \cos^2 x)(\sin^3 x + \cos^3 x)]^2} dx$$

$$= \int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx = \int \frac{\tan^2 x \cdot \sec^2 x}{(1 + \tan^3 x)^2} dx$$

$$\text{Now, put } (1 + \tan^3 x) = t$$

$$\Rightarrow 3 \tan^2 x \sec^2 x dx = dt$$

$$\therefore I = \frac{1}{3} \int \frac{dt}{t^2} = -\frac{1}{3t} + C = \frac{-1}{3(1 + \tan^3 x)} + C$$

26. (a) Let $I = \int \frac{\tan x}{1 + \tan x + \tan^2 x} dx$

$$\Rightarrow I = \int \frac{\tan x + 1 + \tan^2 x}{\tan x + 1 + \tan^2 x} dx - \int \frac{(1 + \tan^2 x)}{1 + \tan x + \tan^2 x}$$

$$\Rightarrow I = x - \int \frac{\sec^2 x dx}{1 + \tan x + \tan^2 x}$$

$$\text{Put } \tan x = t \Rightarrow \sec^2 x \cdot dx = dt$$

$$\therefore I = x - \int \frac{dt}{t^2 + t + \frac{1}{4} + 1 - \frac{1}{4}}$$

$$= x - \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow I = x - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + C$$

$$\Rightarrow I = x - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2 \tan x + 1}{\sqrt{3}} \right) + C$$

$$\therefore A = 3 \text{ and } K = 2$$

27. (b) Suppose, $\frac{x-4}{x+2} = y \Rightarrow x-4 = y(x+2)$

$$\Rightarrow x(1-y) = 2y+4 \Rightarrow x = \frac{2y+4}{1-y}$$

$$\text{So, } f(y) = 2 \left(\frac{2y+4}{1-y} \right) + 1$$

$$\text{Now, } f(x) = 2 \left(\frac{2x+4}{1-x} \right) + 1 = \frac{3x+9}{1-x}$$

$$= \frac{3(x+3)}{1-x} = \frac{3(x-1+4)}{1-x} = -3 + \frac{12}{1-x}$$

$$\therefore \int f(x) dx = -12 \log_e |1-x| - 3x + c$$

28. (a) $\because 7-6x-x^2 = 16-(x+3)^2$

and $\frac{d}{dx} (7-6x-x^2) = -2x-6$

So, $\int \frac{2x+5}{\sqrt{7-6x-x^2}} dx = \int \frac{2x+6}{\sqrt{7-6x-x^2}} dx$

$$- \int \frac{1}{\sqrt{16-(x+3)^2}} dx$$

$$= -2\sqrt{7-6x-x^2} - \sin^{-1}\left(\frac{x+3}{4}\right) + C$$

Therefore, $A = -2$, & $B = -1$.

29. (b) $f\left(\frac{3x-4}{3x+4}\right) = x+2, x \neq -\frac{4}{3}$

Consider $\frac{3x-4}{3x+4} = t$

$$\Rightarrow 3x-4 = 3tx+4t$$

$$\Rightarrow x = \frac{4t+4}{3-3t} + 2$$

$$\Rightarrow f(t) = \frac{10-2t}{3-3t}$$

$$\Rightarrow f(x) = \frac{2x-10}{3x-3}$$

$$\therefore \int f(x) dx = \int \frac{2x-10}{3x-3} dx$$

$$= \int \frac{2x}{3x-3} dx - 10 \int \frac{dx}{3x-3}$$

$$= \frac{2}{3} \int \frac{x-1}{x-1} dx + \frac{2}{3} \int \frac{dx}{x-1} - \frac{10}{3} \int \frac{dx}{x-1}$$

$$= \frac{2x}{3} - \frac{8}{3} \ln(x-1) + C$$

Here, $A = -\frac{8}{3}$, $B = \frac{2}{3}$

$$\therefore (A, B) = \left(-\frac{8}{3}, \frac{2}{3}\right)$$

30. (a) Let, $I = \int \sqrt{1+2\cot x \operatorname{cosec} x + 2\cot^2 x} \cdot dx$

$$\Rightarrow I = \int \sqrt{\frac{\sin^2 x + 2\cos x + 2\cos^2 x}{\sin^2 x}} \cdot dx$$

$$\Rightarrow I = \int \sqrt{\frac{1+2\cos x + \cos^2 x}{\sin x}} \cdot dx$$

$$\Rightarrow I = \int \left| \frac{1+\cos x}{\sin x} \right| \cdot dx$$

$$\Rightarrow I = \int |\operatorname{cosec} x + \cot x| \cdot dx$$

$$\Rightarrow I = \log |\operatorname{cosec} x - \cot x| + \log |\sin x| + C$$

$$\Rightarrow I = \log |1 - \cos x| + C$$

$$\Rightarrow I = \log \left| 2\sin^2 \frac{x}{2} \right| + C$$

$$\Rightarrow I = \log \left| \sin^2 \frac{x}{2} \right| + \log 2 + C$$

$$\Rightarrow I = 2\log \left| \sin \frac{x}{2} \right| + C_1$$

31. (a) $\int \frac{dx}{\cos^3 x \sqrt{4\sin x \cos x}} = \int \frac{dx}{2\cos^4 x \sqrt{\tan x}}$

Let $\tan x = t^2 \Rightarrow \sec^2 x = 1+t^4$

$\sec^2 x dx = 2t dt$

$$= \int \frac{\sec^4 x dx}{2\sqrt{\tan x}} = \int \frac{\sec^2 x (\sec^2 x dx)}{2\sqrt{\tan x}}$$

$$= \int \frac{(1+t^4)2t dt}{2t} = \int (1+t^4) dt = t + \frac{t^5}{5} + k$$

$$= \sqrt{\tan x} + \frac{1}{5} \tan^{5/2} x + k \left[t = \sqrt{\tan x} \right]$$

$$A = \frac{1}{2}, B = \frac{5}{2}, C = \frac{1}{5}$$

$$A+B+C = \frac{16}{5}$$

32. (d) Let $I = \int \frac{\log(t+\sqrt{1+t^2})}{\sqrt{1+t^2}} dt$

put $u = \log(t+\sqrt{1+t^2})$

$$du = \frac{1}{t+\sqrt{1+t^2}} \cdot \left[\frac{t+\sqrt{1+t^2}}{\sqrt{1+t^2}} \right] = \frac{1}{\sqrt{1+t^2}} dt$$

$$\therefore I = \int u du = \frac{u^2}{2} + c$$

Since, $I = \frac{1}{2} [g(t)]^2 + c$

$$\therefore g(t) = \log(t + \sqrt{1+t^2})$$

$$\text{Put } t = 2$$

$$g(2) = \log(2 + \sqrt{5})$$

$$\begin{aligned} 33. \quad (d) \quad \text{Let } I &= \int \left(1 + x - \frac{1}{x}\right) e^{x+1/x} dx \\ &= \int e^{x+1/x} dx + \int \left(x - \frac{1}{x}\right) e^{x+1/x} dx \\ &= x \cdot e^{x+1/x} - \int x \left(1 - \frac{1}{x^2}\right) e^{x+1/x} dx + \int \left(x - \frac{1}{x}\right) e^{x+1/x} dx \\ &= x \cdot e^{x+1/x} - \int \left(x - \frac{1}{x}\right) e^{x+1/x} dx + \int \left(x - \frac{1}{x}\right) e^{x+1/x} dx \\ &= x e^{x+1/x} + C \end{aligned}$$

$$34. \quad (b) \quad \text{Let } I = \int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$$

$$I = \int \left(\frac{\sin x \cos x}{\sin^3 x + \cos^3 x} \right)^2 dx$$

$$I = \int \left(\frac{\sin x \cancel{\cos x}}{\cancel{\cos^3 x} (1 + \tan^3 x)} \right)^2 dx$$

$$= \int \left(\frac{\sin x \sec^2 x}{(1 + \tan^3 x)} \right)^2 dx$$

$$\text{Put } 1 + \tan^3 x = t$$

$$dt = 3 \tan^2 x \sec^2 x dx \quad \text{or } dx = \frac{dt}{3 \tan^2 x \sec^2 x}$$

$$\therefore I = \int \frac{\sin^2 x \sec^4 x}{t^2} \times \frac{dt}{3 \tan^2 x \sec^2 x}$$

$$I = \frac{1}{3} \int \frac{\sin^2 x \sec^4 x}{t^2} \times \frac{dt}{\frac{\sin^2 x}{\cos^2 x} \times \sec^2 x}$$

$$= \frac{1}{3} \int \frac{\cancel{\sin^2 x} \sec^4 x}{t^2} \times \frac{dt}{\cancel{\sin^2 x} \sec^4 x}$$

$$\therefore I = \frac{1}{3} \int \frac{dt}{t^2} = \frac{1}{3} \int t^{-2} dt$$

$$I = \frac{1}{3} \left[\frac{t^{-2+1}}{-2+1} \right] + C = \frac{-1}{3} \left[\frac{1}{t} \right] + C$$

$$\text{or } I = -\frac{1}{3(1 + \tan^3 x)} + C$$

$$35. \quad (a) \quad \text{Let } I = \int x \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx$$

$$\therefore I = 2 \int \frac{x \cdot \tan^{-1} x}{1} dx$$

Applying Integration by parts

$$I = 2 \left[\tan^{-1} x \int x dx - \int \left(\frac{d}{dx} (\tan^{-1} x) \right) \int x dx dx \right]$$

$$I = 2 \left[\frac{x^2}{2} \tan^{-1} x - \int \frac{1}{1+x^2} \times \frac{x^2}{2} dx \right] + C$$

$$I = 2 \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2 + 1 - 1}{x^2 + 1} dx \right] + C$$

$$I = 2 \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{\cancel{x^2} + 1}{\cancel{x^2} + 1} dx + \frac{1}{2} \int \frac{1}{1+x^2} dx \right] + C$$

$$I = 2 \left[\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int 1 dx + \frac{1}{2} \tan^{-1} x \right] + C$$

$$I = 2 \left[\frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x \right] + C$$

$$I = x^2 \tan^{-1} x + \tan^{-1} x - x + C$$

$$\text{or } I = -x + (x^2 + 1) \tan^{-1} x + C$$

$$36. \quad (b) \quad \text{Let } I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2 \sin^2 x \cos^2 x} dx$$

$$= \int \frac{(\sin^4 x)^2 - (\cos^4 x)^2}{1 - 2 \sin^2 x \cos^2 x} dx$$

$$= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{1 - 2 \sin^2 x \cos^2 x} dx$$

$$= \int \frac{[(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x] [(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)]}{1 - 2 \sin^2 x \cos^2 x} dx$$

$$= -\int \cos 2x dx = \frac{-\sin 2x}{2} + C = -\frac{1}{2} \sin 2x + C$$

$$37. \quad (c) \quad \text{Let } \int f(x) dx = \psi(x)$$

$$\text{Let } I = \int x^5 f(x^3) dx$$

$$\text{put } x^3 = t$$

$$\Rightarrow 3x^2 dx = dt$$

$$I = \frac{1}{3} \int 3 \cdot x^2 \cdot x^3 \cdot f(x^3) \cdot dx$$

$$= \frac{1}{3} \int t f(t) dt = \frac{1}{3} \left[t \int f(t) dt - \int f(t) dt \right]$$

$$\begin{aligned}
 &= \frac{1}{3} \left[t\psi(t) - \int \psi(t) dt \right] \\
 &= \frac{1}{3} \left[x^3 \psi(x^3) - 3 \int x^2 \psi(x^3) dx \right] + c \\
 &= \frac{1}{3} x^3 \psi(x^3) - \int x^2 \psi(x^3) dx + c
 \end{aligned}$$

38. (a) Let $I = \int \frac{\cos 8x + 1}{\cot 2x - \tan 2x} dx$

$$\begin{aligned}
 \text{Now, } D' &= \cot 2x - \tan 2x = \frac{\cos 2x}{\sin 2x} - \frac{\sin 2x}{\cos 2x} \\
 &= \frac{\cos^2 2x - \sin^2 2x}{\sin 2x \cos 2x} = \frac{2 \cos 4x}{\sin 4x} \\
 \therefore I &= \int \frac{2 \cos^2 4x}{2 \cos 4x} dx = \int \frac{2 \cos^2 4x \cdot \sin 4x}{2 \cos 4x} dx \\
 &= \frac{1}{2} \int \sin 8x dx = -\frac{1}{2} \cdot \frac{\cos 8x}{8} + k = -\frac{1}{16} \cos 8x + k \\
 \text{Now, } -\frac{1}{16} \cos 8x + k &= A \cos 8x + k \\
 \Rightarrow A &= -\frac{1}{16}
 \end{aligned}$$

39. (d) $\int \frac{5 \tan x}{\tan x - 2} dx = \int \frac{5 \frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} - 2} dx$

$$\begin{aligned}
 &= \int \left(\frac{5 \sin x}{\cos x} \times \frac{\cos x}{\sin x - 2 \cos x} \right) dx \\
 &= \int \frac{5 \sin x dx}{\sin x - 2 \cos x} \\
 &= \int \left(\frac{4 \sin x + \sin x + 2 \cos x - 2 \cos x}{\sin x - 2 \cos x} \right) dx \\
 &= \int \frac{(\sin x - 2 \cos x) + (4 \sin x + 2 \cos x)}{\sin x - 2 \cos x} dx \\
 &= \int \frac{(\sin x - 2 \cos x) + 2(\cos x + 2 \sin x)}{(\sin x - 2 \cos x)} dx \\
 &= \int \frac{\sin x - 2 \cos x}{\sin x - 2 \cos x} dx + 2 \int \frac{(\cos x + 2 \sin x)}{(\sin x - 2 \cos x)} dx \\
 &= \int dx + 2 \int \frac{\cos x + 2 \sin x}{\sin x - 2 \cos x} dx = I_1 + I_2
 \end{aligned}$$

where, $I_1 = \int dx$ and $I_2 = 2 \int \frac{\cos x + 2 \sin x}{\sin x - 2 \cos x} dx$

Let $\sin x - 2 \cos x = t$

$\Rightarrow (\cos x + 2 \sin x) dx = dt$

$\therefore I_2 = 2 \int \frac{dt}{t} = 2 \ln t + C = 2 \ln (\sin x - 2 \cos x) + C$

Hence, $I_1 + I_2 = \int dx + 2 \ln (\sin x - 2 \cos x) + c$

$= x + 2 \ln |(\sin x - 2 \cos x)| + k \Rightarrow a = 2$

40. (a) Let $f(x) = \int \left(\frac{x^2 + \sin^2 x}{1 + x^2} \right) \sec^2 x dx$

$$\begin{aligned}
 &= \int \frac{x^2 \sec^2 x + \frac{\sin^2 x}{\cos^2 x}}{1 + x^2} dx \\
 &= \int \frac{x^2 \sec^2 x + \tan^2 x}{1 + x^2} dx \\
 &= \int \frac{x^2 (1 + \tan^2 x) + \tan^2 x}{1 + x^2} dx \\
 &= \int \frac{x^2 + \tan^2 x (1 + x^2)}{1 + x^2} dx \\
 &= \int \frac{x^2}{1 + x^2} dx + \int \tan^2 x dx \\
 &= \int \frac{x^2 + 1 - 1}{1 + x^2} dx + \int (\sec^2 x - 1) dx \\
 &= \int 1 dx - \int \frac{dx}{1 + x^2} + \int \sec^2 x dx - \int dx \\
 &= -\tan^{-1} x + \tan x + c \\
 \text{Given : } f(0) &= 0 \\
 \Rightarrow f(0) &= -\tan^{-1} 0 + \tan 0 + c \Rightarrow c = 0 \\
 \therefore f(x) &= -\tan^{-1} x + \tan x \\
 \text{Now, } f(1) &= -\tan^{-1}(1) + \tan 1 = \tan 1 - \frac{\pi}{4}
 \end{aligned}$$

41. (a) Let $I = \int \frac{x^2 - x}{x^3 - x^2 + x - 1} dx$

$$\begin{aligned}
 &= \int \frac{x(x-1)}{x^2(x-1) + (x-1)} dx = \int \frac{x dx}{x^2 + 1} = \frac{1}{2} \int \frac{2x dx}{x^2 + 1}
 \end{aligned}$$

Let $x^2 + 1 = t \Rightarrow 2x dx = dt$

$\therefore I = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \log t + c$

$= \frac{1}{2} \log (x^2 + 1) + c$

where 'c' is the constant of integration.

42. (d) Statement - 2: $\cos^3 x$ is a periodic function.

It is a true statement.

Statement - 1

Given $f(x) = \int \cos^3 x dx = \int \left(\frac{\cos 3x}{4} + \frac{3 \cos x}{4} \right) dx$

$= \frac{1}{4} \frac{\sin 3x}{3} + \frac{3}{4} \sin x = \frac{1}{12} \sin 3x + \frac{3}{4} \sin x$

Now, period of $\frac{1}{12} \sin 3x = \frac{2\pi}{3}$

Period of $\frac{3}{4} \sin x = 2\pi$

Hence period of $f(x) = \frac{\text{L.C.M.}(2\pi, 2\pi)}{\text{HCF of } (1, 3)} = \frac{2\pi}{1} = 2\pi$

Thus, $f(x)$ is a periodic function of period 2π .

Hence, Statement - 1 is false.

43. (c) Let $I = \sqrt{2} \int \frac{\sin x dx}{\sin\left(x - \frac{\pi}{4}\right)}$

Let $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$$\Rightarrow I = \sqrt{2} \int \frac{\sin\left(t + \frac{\pi}{4}\right)}{\sin t} dt = \frac{\sqrt{2}}{\sqrt{2}} \int \left(\frac{\sin t + \cos t}{\sin t} \right) dt$$

$$\Rightarrow I = \int (1 + \cot t) dt = t + \log |\sin t| + c_1$$

$$= x - \frac{\pi}{4} + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c_1$$

$$= x + \log \left| \sin \left(x - \frac{\pi}{4} \right) \right| + c \quad \left(\text{where } c = c_1 - \frac{\pi}{4} \right)$$

44. (c) $I = \int \frac{dx}{\cos x + \sqrt{3} \sin x}$

$$\Rightarrow I = \int \frac{dx}{2 \left[\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right]}$$

$$= \frac{1}{2} \int \frac{dx}{\left[\sin \frac{\pi}{6} \cos x + \cos \frac{\pi}{6} \sin x \right]} = \frac{1}{2} \int \frac{dx}{\sin \left(x + \frac{\pi}{6} \right)}$$

$$\Rightarrow I = \frac{1}{2} \int \operatorname{cosec} \left(x + \frac{\pi}{6} \right) dx$$

We know that

$$\int \operatorname{cosec} x dx = \log |(\tan x/2)| + C$$

$$\therefore I = \frac{1}{2} \cdot \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + C$$

45. (a) $\int \frac{dx}{\cos x - \sin x} = \int \frac{dx}{\sqrt{2} \left(\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x \right)}$

$$= \int \frac{dx}{\sqrt{2} \cos \left(x + \frac{\pi}{4} \right)} = \frac{1}{\sqrt{2}} \int \sec \left(x + \frac{\pi}{4} \right) dx$$

$$= \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} + \frac{\pi}{8} \right) \right| + C$$

$$\left[\because \int \sec x dx = \log \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| \right]$$

$$= \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + C$$

46. (b) $\int \frac{\sin x}{\sin(x-\alpha)} dx = \int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)} dx$

$$= \int \frac{\sin(x-\alpha) \cos \alpha + \cos(x-\alpha) \sin \alpha}{\sin(x-\alpha)} dx$$

$$= \int \{ \cos \alpha + \sin \alpha \cot(x-\alpha) \} dx$$

$$= (\cos \alpha)x + (\sin \alpha) \log \sin(x-\alpha) + C$$

Comparing with $Ax + B \log \sin(x-\alpha) + c$

$$\therefore A = \cos \alpha, B = \sin \alpha$$

47. (d) $\therefore f'''(x) - g''(x) = 0$

Integrating, $f'(x) - g'(x) = c$;

$$\Rightarrow f'(1) - g'(1) = c \Rightarrow 4 - 2 = c \Rightarrow c = 2.$$

$$\therefore f'(x) - g'(x) = 2;$$

Integrating, $f(x) - g(x) = 2x + c_1$

$$\Rightarrow f(2) - g(2) = 4 + c_1 \Rightarrow 9 - 3 = 4 + c_1;$$

$$\Rightarrow c_1 = 2 \therefore f(x) - g(x) = 2x + 2$$

$$\text{At } x = 3/2, f(x) - g(x) = 3 + 2 = 5.$$

48. (c) $I = \int_1^2 e^x x^x (2 + \log_e x) dx$

$$I = \int_1^2 e^x x^x [1 + (1 + \log_e x)] dx$$

$$= \int_1^2 e^x [x^x + x^x (1 + \log_e x)] dx$$

$$\because \int e^x (f(x) + f'(x)) dx = e^x f(x) + c$$

$$\therefore I = \left[e^x x^x \right]_1^2$$

$$= e^2 \times 4 - e \times 1 = 4e^2 - e = e(4e - 1)$$

49. (a) $\int_{\alpha}^{\alpha+1} \frac{dx}{(x+\alpha)(x+\alpha+1)}$

$$= \int_{\alpha}^{\alpha+1} \left[\frac{1}{x+\alpha} - \frac{1}{x+\alpha+1} \right] dx \quad [\text{Using partial fraction}]$$

$$= \log \left(\frac{(x+\alpha)}{(x+\alpha+1)} \right) \Big|_{\alpha}^{\alpha+1} = \log \left(\frac{2\alpha+1}{2\alpha+2} \cdot \frac{2\alpha+1}{2\alpha} \right)$$

$$= \log \frac{9}{8} \quad (\text{Given})$$

$$\text{So, } \frac{(2\alpha+1)^2}{\alpha(\alpha+1)} = \frac{9}{2} \Rightarrow 8\alpha^2 + 8\alpha + 2 = 9\alpha^2 + 9\alpha$$

$$\Rightarrow \alpha^2 + \alpha - 2 = 0 \Rightarrow \alpha = 1, -2$$

50. (c) Let, $1 = \int x^2 \cdot e^{-x^2} dx$

$$\text{Put } -x^2 = t \Rightarrow -2x dx = dt$$

$$1 = \int \frac{t^2 \cdot e^t dt}{(-2)} = \frac{-1}{2} e^t (t^2 - 2t + 2) c$$

$$\therefore g(x) = \frac{-1}{2} (x^4 + 2x^2 + 2) \Rightarrow g(-1) = \frac{-5}{2}$$

51. (b) $I = \int x^5 e^{-4x^3} dx$

$$\text{Put } -4x^3 = \theta$$

$$\Rightarrow -12x^2 dx = d\theta$$

$$\Rightarrow x^2 dx = -\frac{d\theta}{12}$$

$$I = \int \frac{1}{48} \theta e^\theta d\theta = \frac{1}{48} [\theta e^\theta - e^\theta] + C$$

$$I = \frac{1}{48} e^{-4x^3} (-4x^3 - 1) + C$$

Then, by comparison

$$f(x) = -4x^3 - 1$$

52. (c) $I = \int \frac{dx}{(1+\sqrt{x}) \cdot \sqrt{x} \sqrt{1-x}}$

$$\text{Put } 1 + \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow I = \int \frac{2dt}{t\sqrt{2t-t^2}}$$

$$\text{Again put } t = \frac{1}{z} \Rightarrow dt = \frac{-1}{z^2} dz$$

$$\Rightarrow I = 2 \int \frac{\frac{-1}{z^2} dz}{\frac{1}{z} \sqrt{2 - \frac{1}{z^2}}} = 2 \int \frac{-dz}{\sqrt{2z-1}}$$

$$= -2\sqrt{2z-1} + c = -2\sqrt{\frac{2}{t}-1} + c$$

$$= -2\sqrt{\frac{2-t}{t}} + c = -2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} + c$$

53. (b) $I = \int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^3(1+x^{-4})^{3/4}}$

$$\text{Let } x^{-4} = y$$

$$\Rightarrow -4x^{-5} dx = dy \Rightarrow dx = \frac{-1}{4} x^3 dy$$

$$\therefore I = \frac{-1}{4} \int \frac{x^3 dy}{x^3(1+y)^{3/4}} = \frac{-1}{4} \int \frac{dy}{(1+y)^{3/4}}$$

$$= \frac{-1}{4} \times 4(1+y)^{1/4} = -(1+x^{-4})^{1/4} + C$$

$$= -\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$$

54. (b) $\int \frac{dx}{(x+1)^{3/4}(x-2)^{5/4}}$

$$\int \frac{dx}{\left(\frac{x+1}{x-2}\right)^{3/4} (x-2)^2}, \quad \text{put } \frac{x+1}{x-2} = t$$

$$\frac{-3}{(x-2)^2} = \frac{dt}{dx}$$

$$\frac{dx}{(x-2)^2} = -\frac{dt}{3} = \frac{-1}{3} \int \frac{dt}{t^{3/4}} = -\frac{1}{3} \int t^{-3/4} dt$$

$$= \frac{1}{3} \left[\frac{-3+1}{t^{-3/4+1}} \right] = \frac{-4}{3} \left[\frac{x+1}{x-2} \right]^{1/4} + c$$

55. (b) $\int \frac{x^{5m-1} + 2x^{4m-1}}{(x^{2m} + x^m + 1)^3} dx = \int \frac{x^{5m-1} + 2x^{4m-1}}{x^{6m}(1+x^{-m}+x^{-2m})^3} dx$

$$= \int \frac{x^{-m-1} + 2x^{-2m-1}}{(1+x^{-m}+x^{-2m})^3} dx$$

$$\text{Put } t = 1 + x^{-m} + x^{-2m}$$

$$\therefore \frac{dt}{dx} = -mx^{-m-1} - 2mx^{-2m-1}$$

$$\Rightarrow \frac{dt}{-m} = (x^{-m-1} + 2x^{-2m-1}) dx$$

$$\therefore \int \frac{x^{5m-1} + 2x^{4m-1}}{(x^{2m} + x^m + 1)^3} dx = \frac{1}{-m} \int t^{-3} dt = \frac{1}{2mt^2} + C$$

$$= \frac{1}{2m(1+x^{-m}+x^{-2m})^2} + C$$

$$= \frac{x^{4m}}{2m(x^{2m} + x^m + 1)^2} + C$$

$$\therefore f(x) = \frac{x^{4m}}{2m(x^{2m} + x^m + 1)^2}$$

56. (b) $I = \int \frac{x dx}{2-x^2+\sqrt{2-x^2}}$

$$\text{Put } t = \sqrt{2-x^2}, \quad \frac{dt}{dx} = \frac{1}{2\sqrt{2-x^2}} \cdot (-2x)$$

$$\Rightarrow -t \, dt = x \, dx$$

$$\therefore I = \int \frac{(-t) \, dt}{t^2 + t} = - \int \frac{1}{t+1} \, dt = -\log |t+1|$$

$$= -\log \left| \sqrt{2-x^2} + 1 \right| + c$$

57. (b) Let $I = \int \frac{x^2 - x + 1}{x^2 + 1} \cdot e^{\cot^{-1} x} \, dx$

Put $x = \cot t \Rightarrow -\operatorname{cosec}^2 t \, dt = dx$

Now, $1 + \cot^2 t = \operatorname{cosec}^2 t$

$$\therefore I = \int \frac{e^t (\cot^2 t - \cot t + 1)}{(1 + \cot^2 t)} (-\operatorname{cosec}^2 t) \, dt$$

$$= - \int e^t (\operatorname{cosec}^2 t - \cot t) \, dt$$

$$= \int e^t (\cot t - \operatorname{cosec}^2 t) \, dt = e^t \cot t + C$$

$$= e^{\cot^{-1} x} (x) + C \equiv A(x) \cdot e^{\cot^{-1} x} + C$$

$$\Rightarrow A(x) = x$$

58. (a) $\int \frac{x^6}{x+x^7} \, dx = \int \frac{x^6}{x(1+x^6)} \, dx = \int \frac{(1+x^6)-1}{x(1+x^6)} \, dx$

$$= \int \frac{1}{x} \, dx - \int \frac{1}{x+x^7} \, dx = \ln |x| - p(x) + c$$

59. (d) $\int \frac{(\log x - 1)^2}{(1 + (\log x)^2)^2} \, dx = \int \frac{1 + (\log x)^2 - 2 \log x}{[1 + (\log x)^2]^2} \, dx$

$$= \int \left[\frac{1}{(1 + (\log x)^2)} - \frac{2 \log x}{(1 + (\log x)^2)^2} \right] \, dx$$

$$\therefore I = \int \left[\frac{e^t}{1+t^2} - \frac{2t e^t}{(1+t^2)^2} \right] \, dt$$

$$= \int e^t \left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2} \right] \, dt$$

[Which is of the form $\int e^x (f(x) + f'(x)) \, dx$

$$= f(x) \cdot e^x + c]$$

$$= \frac{e^t}{1+t^2} + c = \frac{x}{1 + (\log x)^2} + c$$

60. (c) $I_2 = \int_0^1 (1-x^{50})^{101} \, dx = \int_0^1 (1-x^{50})(1-x^{50})^{100} \, dx$

$$I_2 = \int_0^1 (1-x^{50})^{100} \, dx - \int_0^1 x^{49} \underbrace{(1-x^{50})^{100}}_I \, dx$$

$$I_2 = I_1 + \left[\frac{x}{5050} (1-x^{50})^{101} \right]_0^1 - \int_0^1 \frac{(1-x^{50})^{101}}{5050} \, dx$$

$$I_2 = I_1 + 0 - \frac{I_2}{5050}$$

$$\Rightarrow \frac{5051}{5050} I_2 = I_1 \Rightarrow I_2 = \frac{5050}{5051} I_1$$

$$\Rightarrow \alpha = \frac{5050}{5051}$$

61. (c) $I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}} \, dx$

$$= \int_{-\pi/2}^0 \frac{1}{1+e^{\sin x}} \, dx + \int_0^{\pi/2} \frac{1}{1+e^{\sin x}} \, dx$$

$$= \int_0^{\pi/2} \left(\frac{1}{1+e^{\sin x}} + \frac{1}{1+e^{-\sin x}} \right) \, dx$$

$$= \int_0^{\pi/2} \frac{1+e^{\sin x}}{1+e^{\sin x}} \, dx = \frac{\pi}{2}$$

62. (a) $f(x) = |x-2| = \begin{cases} 2-x, & x < 2 \\ x-2, & x \geq 2 \end{cases}$

$$g(x) = f(f(x)) = \begin{cases} 2-f(x), & f(x) < 2 \\ f(x)-2, & f(x) \geq 2 \end{cases}$$

$$= \begin{cases} 2-(2-x), & 2-x < 2, & x < 2 \\ (2-x)-2, & 2-x \geq 2, & x < 2 \\ 2-(x-2), & x-2 < 2, & x \geq 2 \\ (x-2)-2, & x-2 \geq 2, & x \geq 2 \end{cases}$$

$$= \begin{cases} -x & 0 < x \leq 0 \\ x & 0 < x < 2 \\ 4-x & 2 \leq x < 4 \\ x-4 & x \geq 4 \end{cases}$$

$$\therefore \int_0^3 [g(x) - f(x)] \, dx$$

$$= \int_0^2 x \, dx + \int_2^3 (4-x) \, dx - \int_0^3 |x-2| \, dx = 1$$

63. (c) $\int_{\pi/6}^{\pi/3} \left[\frac{1}{2} \frac{d(\tan^4 x)}{dx} \cdot \sin^4 3x + \frac{1}{2} \tan^4 x \cdot \frac{d(\sin^4 3x)}{dx} \right] \, dx$

$$= \frac{1}{2} \int_{\pi/6}^{\pi/3} d(\tan^4 x \cdot \sin^4 3x) \, dx$$

$$= \left[\frac{\tan^4 x \sin^4 3x}{2} \right]_{\pi/6}^{\pi/3} = \frac{9 \cdot 0}{2} - \frac{\frac{1}{9} \cdot 1}{2} = -\frac{1}{18}$$

64. (21)

$$\int_0^n \{x\} \, dx = n \int_0^1 x \, dx = \frac{n}{2}$$

$$\Rightarrow \int_0^n [x] dx = \int_0^n (x - \{x\}) dx = \frac{n^2}{2} - \frac{n}{2}$$

According to the questions,

$$\frac{n}{2}, \frac{n^2 - n}{2}, 10(n^2 - n) \text{ are in GP}$$

$$\therefore \left(\frac{n^2 - n}{2} \right)^2 = \frac{n}{2} \times 10(n^2 - n)$$

$$\Rightarrow n^2 = 21n \Rightarrow n = 21.$$

65. (c) $I = \int_{-\pi}^{\pi} |\pi - |x|| dx \quad [\because |\pi - |x|| \text{ is even}]$

$$= 2 \int_0^{\pi} |\pi - x| dx$$

$$= 2 \int_0^{\pi} (\pi - x) dx$$

$$= 2 \left[\pi x - \frac{x^2}{2} \right]_0^{\pi} = 2 \left(\pi^2 - \frac{\pi^2}{2} \right) = \pi^2.$$

66. (a) $\frac{k}{6} = \int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}} dx$

Let $x = \sin \theta$; $dx = \cos \theta d\theta$,

$$\text{then } \int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{3/2}} dx = \int_0^{\frac{\pi}{6}} \frac{\sin^2 \theta \cos \theta}{\cos^3 \theta} d\theta$$

$$\therefore \frac{k}{6} = \int_0^{\frac{\pi}{6}} \frac{\sin^2 \theta}{\cos^3 \theta} \cdot \cos \theta d\theta$$

$$\Rightarrow \frac{k}{6} = \int_0^{\frac{\pi}{6}} \tan^2 \theta d\theta = \int_0^{\frac{\pi}{6}} (\sec^2 \theta - 1) d\theta$$

$$\Rightarrow \frac{k}{6} = (\tan \theta - \theta)_0^{\pi/6} = \left(\frac{1}{\sqrt{3}} - \frac{\pi}{6} \right) = \frac{2\sqrt{3} - \pi}{6}$$

$$\Rightarrow k = 2\sqrt{3} - \pi$$

67. (1.50)

$$\int_0^2 \|x-1|-x| dx = \int_0^1 |1-x-x| dx + \int_1^2 |x-1-x| dx$$

$$= \int_0^1 (1-2x) dx + \int_{1/2}^1 (2x-1) dx + \int_1^2 dx$$

$$= [x - x^2]_0^{\frac{1}{2}} + [x^2 - x]_1^1 + [x]_1^2$$

$$= \frac{1}{2} - \frac{1}{4} + (1-1) - \left(\frac{1}{4} - \frac{1}{2} \right) + 2-1 = \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$$

68. (1)

$$\int_1^2 |2x - [3x]| dx$$

$$= \int_1^2 |3x - [3x] - x| dx$$

$$= \int_1^2 |\{3x\} - x| dx = \int_1^2 (x - \{3x\}) dx$$

$$= \int_1^2 x dx - \int_1^2 \{3x\} dx$$

$$= \left[\frac{x^2}{2} \right]_1^2 - 3 \int_0^{1/3} 3x dx$$

$$= \frac{(4-1)}{2} - 9 \left[\frac{x^2}{2} \right]_0^{1/3} = \frac{3}{2} - \frac{1}{2} = 1$$

69. (d) $\int_0^1 (a + bx + cx^2) dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3} \Big|_0^1 = a + \frac{b}{2} + \frac{c}{3}$

Now, $f(1) = a + b + c$, $f(0) = a$ and $f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$

Now, $\frac{1}{6} \left(f(1) + f(0) + 4f\left(\frac{1}{2}\right) \right)$

$$= \frac{1}{6} \left(a + b + c + a + 4 \left(a + \frac{b}{2} + \frac{c}{4} \right) \right)$$

$$= \frac{1}{6} (6a + 3b + 2c) = a + \frac{b}{2} + \frac{c}{3}$$

Hence, $\int_0^1 f(x) dx = \frac{1}{6} \left\{ f(0) + f(1) + 4f\left(\frac{1}{2}\right) \right\}$

70. (c) $\int_0^{2\pi} \frac{x \sin^8 x}{\sin^8 x + \cos^8 x} dx$

$$= \int_0^{\pi} \left[\frac{x \sin^8 x}{\sin^8 x + \cos^8 x} + \frac{(2\pi - x) \sin^8 x}{\sin^8 x + \cos^8 x} \right] dx$$

$$\left[\because \int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_a^{2a} f(2a-x) dx \right]$$

$$= \int_0^{\pi} \frac{2\pi \sin^8 x}{\sin^8 x + \cos^8 x} dx$$

$$= 2\pi \int_0^{\pi/2} \left[\frac{\sin^8 x}{\sin^8 x + \cos^8 x} + \frac{\cos^8 x}{\sin^8 x + \cos^8 x} \right] dx$$

$$= 2\pi \int_0^{\pi/2} 1 dx = 2\pi \times \frac{\pi}{2} = \pi^2$$

71. (b) $f(x) = \frac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$

$$f'(x) = \frac{-1}{2} \left(\frac{(6x^2 - 18x + 12)}{(2x^3 - 9x^2 + 12x + 4)^{3/2}} \right)$$

$$= \frac{-6(x-1)(x-2)}{2(2x^3 - 9x^2 + 12x + 4)^{3/2}}$$

$$f(1) = \frac{1}{3} \text{ and } f(2) = \frac{1}{\sqrt{8}}$$

It is increasing function

$$\frac{1}{3} < I < \frac{1}{\sqrt{8}}$$

$$\frac{1}{9} < I^2 < \frac{1}{8}$$

72. (c) $I = \frac{1}{(a+b)} \int_a^b x[f(x) + f(x+1)] dx \quad \dots(i)$

$$x \rightarrow a+b-x$$

$$I = \frac{1}{(a+b)} \int_a^b (a+b-x)[f(a+b-x) + f(a+b+1-x)] dx$$

$$I = \frac{1}{(a+b)} \int_a^b (a+b-x)[f(x+1) + f(x)] dx \quad \dots(ii)$$

$$[\because \text{put } x \rightarrow x+1 \text{ in } f(a+b+1-x) = f(x)]$$

Add (i) and (ii)

$$2I = \int_a^b [f(x+1) + f(x)] dx$$

$$2I = \int_a^b f(x+1) dx + \int_a^b f(x) dx$$

$$= \int_a^b f(a+b+1-x) dx + \int_a^b f(x) dx$$

$$2I = 2 \int_a^b f(x) dx$$

$$\therefore \int_{a-1}^{b-1} f(x+1) dx \quad [\because \text{Put } x \rightarrow x+1]$$

73. (a) $4\alpha \left\{ \int_{-1}^0 e^{\alpha x} dx + \int_0^2 e^{-\alpha x} dx \right\} = 5$

$$\Rightarrow 4\alpha \left\{ \frac{e^{\alpha x}}{\alpha} \Big|_{-1}^0 + \frac{e^{-\alpha x}}{-\alpha} \Big|_0^2 \right\} = 5$$

$$\Rightarrow 4\alpha \left\{ \left(\frac{1-e^{-\alpha}}{\alpha} \right) - \left(\frac{e^{-2\alpha}-1}{\alpha} \right) \right\} = 5$$

$$\Rightarrow 4(2-e^{-\alpha}-e^{-2\alpha}) = 5$$

$$\text{Put } e^{-\alpha} = t$$

$$\Rightarrow 4t^2 + 4t - 3 = 0 \quad \Rightarrow (2t+3)(2t-1) = 0$$

$$\Rightarrow e^{-\alpha} = \frac{1}{2} \Rightarrow \alpha = \log_e 2$$

74. (a) $2\cot^2 \theta - \frac{5}{\sin \theta} + 4 = 0$

$$\frac{2\cos^2 \theta}{\sin^2 \theta} - \frac{5}{\sin \theta} + 4 = 0$$

$$\Rightarrow 2\cos^2 \theta - 5\sin \theta + 4\sin^2 \theta = 0, \sin \theta \neq 0$$

$$\Rightarrow 2\sin^2 \theta - 5\sin \theta + 2 = 0$$

$$\Rightarrow (2\sin \theta - 1)(\sin \theta - 2) = 0$$

$$\therefore \sin \theta = \frac{1}{2} \quad \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\therefore \int_{\pi/6}^{5\pi/6} \cos^2 3\theta d\theta = \int_{\pi/6}^{5\pi/6} \frac{1+\cos 6\theta}{2} d\theta$$

$$= \frac{1}{2} \left[\theta + \frac{\sin 6\theta}{6} \right]_{\pi/6}^{5\pi/6} = \frac{1}{2} \left[\frac{5\pi}{6} - \frac{\pi}{6} + \frac{1}{6}(0-0) \right]$$

$$= \frac{1}{2} \cdot \frac{4\pi}{6} = \frac{\pi}{3}$$

75. (a) Given, $\int_6^{f(x)} 4t^3 dt = (x-2)g(x)$

Differentiating both sides,

$$4(f(x))^3 \cdot f'(x) = g'(x)(x-2) + g(x)$$

Putting $x = 2$, $\frac{4(6)^3 \cdot 1}{48} = g(2) \Rightarrow \lim_{x \rightarrow 2} g(x) = 18$

$$\begin{aligned}
 76. \quad (d) \quad & \int_0^{\pi/2} \frac{\cot x \, dx}{\cot x + \operatorname{cosec} x} \\
 &= \int_0^{\pi/2} \frac{\cot x \, dx}{1 + \cos x} = \int_0^{\pi/2} \left(1 - \frac{1}{1 + \cos x}\right) dx \\
 &= [x]_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2 \cos^2 \frac{x}{2}} dx = \frac{\pi}{2} - \frac{1}{2} \int_0^{\pi/2} \sec^2 \frac{x}{2} dx \\
 &= \frac{\pi}{2} - \left(\tan \frac{x}{2}\right)_0^{\pi/2} = \frac{\pi}{2} - [1] = \left(\frac{\pi}{2} - 1\right) = m\pi + mn \\
 \therefore m &=, n = -2, \text{ Hence, } mn = -1
 \end{aligned}$$

$$77. \quad (b) \quad I = \int_0^{2\pi} [\sin 2x(1 + \cos 3x)] dx \quad \dots(1)$$

$$\begin{aligned}
 \therefore \int_0^a f(x) dx &= \int_0^a f(a-x) dx \\
 \therefore I &= \int_0^{2\pi} [-\sin 2x(1 + \cos 3x)] dx \quad \dots(2)
 \end{aligned}$$

From (1) + (2), we get;

$$2I = \int_0^{2\pi} (-1) dx \Rightarrow 2I = -(x)_0^{2\pi} \Rightarrow I = -\pi$$

$$78. \quad (c) \quad \text{Let, } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^3 x \cdot \operatorname{cosec}^4 x \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1 \, dx}{\frac{2}{\cos^3 x} \cdot \frac{4}{\sin^3 x}}$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1 \, dx}{\cos^2 x \cdot \tan^3 x} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x \, dx}{\tan^3 x}$$

Let $\tan x = u$

$$\begin{aligned}
 I &= \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} u^{-\frac{4}{3}} du = \frac{3 \left[u^{-\frac{1}{3}} \right]_{\frac{1}{\sqrt{3}}}^{\sqrt{3}}}{-\frac{1}{3}} \\
 &= -3 \left[3^{\frac{1}{6}} - \frac{1}{3^{\frac{1}{6}}} \right] = -3 \left(3^{\frac{1}{6}} - 3^{-\frac{1}{6}} \right) \\
 &= 3 \left(3^{\frac{1}{6}} - 3^{-\frac{1}{6}} \right) = 3 \left(3^{\frac{1}{6}} - \frac{1}{3^{\frac{1}{6}}} \right)
 \end{aligned}$$

$$79. \quad (b) \quad \text{Let } I = \int_0^{\pi/2} \frac{\sin^3 x \, dx}{\sin x + \cos x} \quad \dots(1)$$

Use the property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$$I = \int_0^{\pi/2} \frac{\cos^3 x \, dx}{\sin x + \cos x} \quad \dots(2)$$

Adding equation (1) and (2), we get

$$\Rightarrow 2I = \int_0^{\pi/2} \left(1 - \frac{1}{2} \sin(2x)\right) dx$$

$$\Rightarrow I = \frac{1}{2} \left[x + \frac{1}{4} \cos 2x \right]_0^{\pi/2}$$

$$\Rightarrow I = \frac{\pi - 1}{4}$$

$$80. \quad (a) \quad \int_0^1 x \cot^{-1}(1 - x^2 + x^4) dx = \int_0^1 x \tan^{-1} \left(\frac{1}{1 + x^4 - x^2} \right) dx$$

$$= \int_0^1 x \tan^{-1} \left(\frac{x^2 - (x^2 - 1)}{1 + x^2(x^2 - 1)} \right) dx$$

$$= \frac{1}{2} \int_0^1 \tan^{-1} t^2 dt - \frac{1}{2} \int_{-1}^0 \tan^{-1} k \, dk$$

Put $x^2 = t \Rightarrow 2x dx = dt$ in the first integral

and $x^2 - 1 = k \Rightarrow 2x dx = dk$ in the second integral.

$$= \frac{1}{2} \int_0^1 \tan^{-1} t \, dt - \frac{1}{2} \int_0^1 \tan^{-1} k \, dk$$

$$= \frac{1}{2} \left(t \tan^{-1} t \Big|_0^1 - \int_0^1 \frac{t}{1+t^2} dt \right)$$

$$- \frac{1}{2} \left(k \tan^{-1} k \Big|_0^1 - \int_{-1}^0 \frac{k}{1+k^2} dk \right)$$

$$= \frac{1}{2} \left(\frac{\pi}{4} - \left(\frac{1}{2} \ln(1+t^2) \right) \Big|_0^1 \right) - \frac{1}{2} \left(-\frac{\pi}{4} - \left(\frac{1}{2} \ln(1+k^2) \right) \Big|_{-1}^0 \right)$$

$$= \left(\frac{\pi}{8} - \frac{1}{4} \ln 2 \right) - \left(-\frac{\pi}{8} - \frac{1}{4} \ln 2 \right) = \frac{\pi}{4} - \frac{1}{2} \ln 2$$

81. (d) Using L' Hospital rule and Leibnitz theorem, we get

$$\lim_{x \rightarrow 2} \frac{\int_2^{f(x)} 2t dt}{(x-2)} = \lim_{x \rightarrow 2} \frac{2f(x)f'(x) - 0}{1}$$

$$\text{Putting } x=2, 2f(2)f'(2) = 12f'(2) \quad [\because f(2) = 6]$$

82. (d) $g(f(x)) = \log\left(\frac{2-x\cos x}{2+x\cos x}\right), x > 0$

$$\text{Let } I = \int_{-\pi/4}^{\pi/4} \log\left(\frac{2-x\cos x}{2+x\cos x}\right) dx \quad \dots(i)$$

$$\text{Use the property } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

Then, equation (i) becomes,

$$I = \int_{-\pi/4}^{\pi/4} \log\left(\frac{2+x\cos x}{2-x\cos x}\right) dx \quad \dots(ii)$$

Adding (i) and (ii)

$$2I = \int_{-\pi/4}^{\pi/4} \log\left(\frac{2-x\cos x}{2+x\cos x} \cdot \frac{2+x\cos x}{2-x\cos x}\right) dx$$

$$2I = \int_{-\pi/2}^{\pi/2} \log(1) dx = 0$$

$$\Rightarrow I = 0 = \log 1$$

83. (a) $f(x) = \int_0^x g(g) dt, \quad \dots(i)$

$\because g$ is a non-zero even function.

$$\therefore g(-x) = g(x), \quad \dots(ii)$$

$$\text{Given, } f(x+5) = g(x) \quad \dots(iii)$$

$$\text{From (i) } f'(x) = g(x)$$

$$\text{Let, } I = \int_0^x f(t) dt,$$

$$\text{Put } t = \lambda - 5 \Rightarrow I = \int_5^{x+5} f(\lambda - 5) d\lambda$$

$$\because f(x+5) = g(x)$$

$$\Rightarrow f(-x+5) = g(-x) = g(x) \quad \dots(iv)$$

$$I = \int_5^{x+5} f(\lambda - 5) d\lambda$$

$$f(0) = 0, g(x) \text{ is even} \Rightarrow f(x) \text{ is odd}$$

$$\therefore I = \int_5^{x+5} -f(5-\lambda) d\lambda$$

$$\Rightarrow I = \int_5^{x+5} g(\lambda) d\lambda = \int_{x+5}^5 g(t) dt \quad (\text{from (iv)})$$

84. (c) $f(x) = f(a-x)$

$$g(x) + g(a-x) = 4$$

Let, the integral,

$$I = \int_0^a f(x)g(x) dx$$

$$= \int_0^a f(a-x) \cdot g(a-x) dx$$

$$\left[\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

$$\Rightarrow I = \int_0^a f(x)[4 - g(x)] dx$$

$$\Rightarrow I = \int_0^a 4f(x) dx - \int_0^a f(x) \cdot g(x) dx$$

$$\Rightarrow I = \int_0^a 4f(x) dx - I$$

$$\Rightarrow 2I = \int_0^a 4f(x) dx$$

$$\Rightarrow I = 2 \int_0^a f(x) dx$$

85. (d) $I = \int_1^e \left\{ \left(\frac{x}{e} \right)^{2x} - \left(\frac{e}{x} \right)^x \right\} \log_e x dx$

$$\text{Let } \left(\frac{x}{e} \right)^x = t$$

$$\Rightarrow x \ln \left(\frac{x}{e} \right) = \ln t$$

$$\Rightarrow x (\ln x - 1) = \ln t$$

On differentiating both sides w.r. to x we get

$$\ln x \cdot dx = \frac{dt}{t}$$

When $x = e$ then $t = 1$ and when $x = 1$ then $t = \frac{1}{e}$.

$$I = \int_{\frac{1}{e}}^1 \left(t^2 - \frac{1}{t} \right) \cdot \frac{dt}{t} = \int_{\frac{1}{e}}^1 \left(t - \frac{1}{t^2} \right) dt$$

$$= \left(\frac{t^2}{2} + \frac{1}{t} \right) \Big|_{\frac{1}{e}}^1 = \left(\frac{1}{2} + 1 \right) - \left(\frac{1}{2e^2} + e \right) = \frac{3}{2} - e - \frac{1}{2e^2}$$

86. (a) Let $f(x) = \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}}$

So, $f(-x) = \frac{\sin^2(-x)}{\left[\frac{-x}{\pi}\right] + \frac{1}{2}} \quad \because [-x] = -1 - [x]$

$\Rightarrow f(-x) = \frac{\sin^2 x}{-1 - \left[\frac{x}{\pi}\right] + \frac{1}{2}} = \frac{\sin^2 x}{-\frac{1}{2} - \left[\frac{x}{\pi}\right]} = -f(x)$

$\Rightarrow f(x)$ is odd function

Hence, $\int_{-2}^2 f(x) dx = 0$

87. (b) $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{dx}{\sin 2x (\tan^5 x + \cot^5 x)}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan^5 x \cdot \sec^2 x}{2 \sin x \left((\tan^5 x)^2 + 1 \right)} dx$

$= \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\tan^4 x \cdot \sec^2 x}{(\tan^5 x)^2 + 1} dx$

Let $\tan^4 x = t$.

$5 \tan^4 x \cdot \sec^2 x dx = dt$.

When $x \rightarrow \frac{\pi}{4}$ then $t \rightarrow 1$

and $x \rightarrow \frac{\pi}{6}$ then $t \rightarrow \left(\frac{1}{\sqrt{3}}\right)^5$

$\therefore I = \frac{1}{10} \int_{\left(\frac{1}{\sqrt{3}}\right)^5}^1 \frac{dt}{t^2 + 1}$

$= \frac{1}{10} \left(\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{9\sqrt{3}} \right) \right)$

88. (d) $I = \int_a^b (x^4 - 2x^2) dx$

$\Rightarrow \frac{dI}{dx} = x^4 - 2x^2 = 0$ (for minimum)

$\Rightarrow x = 0, \pm\sqrt{2}$

Also, $I = \left[\frac{x^5}{5} - \frac{2x^3}{3} \right]_a^b$

For $a = 0, b = \sqrt{2}$

$I = \frac{-8\sqrt{2}}{15}$

For $a = -\sqrt{2}, b = 0$

$I = \frac{-8\sqrt{2}}{15}$

For $a = \sqrt{2}, b = -\sqrt{2}$

$I = \frac{16\sqrt{2}}{15}$

For $a = -\sqrt{2}, b = \sqrt{2}$

$I = \frac{-16\sqrt{2}}{15}$

$\therefore I$ is minimum when $(a, b) = (-\sqrt{2}, \sqrt{2})$

89. (a) $\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt$

$\Rightarrow f(x) = 2x - x^2 f(x)$

$\Rightarrow f(x) = \frac{2x}{1+x^2}$

$\Rightarrow f'(x) = \frac{2(1-x^2)}{(1+x^2)^2}$

Then,

$f'(1/2) = \frac{2\left(1 - \frac{1}{4}\right)}{\left(1 + \frac{1}{4}\right)^2} = \frac{3}{2} \times \frac{16}{25} = \frac{24}{25}$

90. (c) $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x] + [\sin x] + 4}$

$= \int_{-\frac{\pi}{2}}^{-1} \frac{dx}{-2-1+4} + \int_{-1}^0 \frac{dx}{-1-1+4} + \int_0^1 \frac{dx}{0+0+4} + \int_1^{\frac{\pi}{2}} \frac{dx}{1+0+4}$

$= \left(-1 + \frac{\pi}{2}\right) + \frac{1}{2}(0+1) + \frac{1}{4}(1-0) + \frac{1}{5}\left(\frac{\pi}{2}-1\right)$

$= \frac{3\pi}{5} - \frac{9}{20} = \frac{3}{20}(4\pi-3)$

91. (b) $I = \int_0^{\pi} |\cos x|^3 dx$

$$= 2 \int_0^{\pi/2} \cos^3 x dx$$

$$= \frac{2}{4} \int_0^{\pi/2} (3 \cos x + \cos 3x) dx$$

$$[\because \cos 3\theta = 4\cos^3 \theta - 3\cos \theta]$$

$$= \frac{1}{2} \left[3 \sin x + \frac{\sin 3x}{3} \right]_0^{\pi/2}$$

$$= \frac{1}{2} \left(3 - \frac{1}{3} \right) = \frac{4}{3}$$

92. (a) $\because f: R \rightarrow R$

$$\text{and } |f(x) - f(y)| \leq 2 \cdot |x - y|^{3/2}$$

$$\Rightarrow \left| \frac{f(x) - f(y)}{x - y} \right| \leq 2\sqrt{x - y}$$

$$\Rightarrow \lim_{x \rightarrow y} \left| \frac{f(x) - f(y)}{x - y} \right| \leq \lim_{x \rightarrow y} 2\sqrt{x - y}$$

$$\Rightarrow |f'(x)| = 0$$

$$\therefore f(x) \text{ is a constant function.}$$

$$\text{Given } f(0) = 1 \Rightarrow f(x) = 1$$

Hence, the integral

$$\int_0^1 f^2(x) dx = \int_0^1 1 dx = [x]_0^1 = 1$$

93. (d) Let, $I = \int_0^{\pi/3} \frac{\tan \theta}{\sqrt{2k \sec \theta}} d\theta$

$$= \frac{1}{\sqrt{2k}} \int_0^{\pi/3} \frac{\sin \theta}{\sqrt{\cos \theta}} d\theta$$

$$\text{Let } \cos \theta = t^2$$

$$\therefore \sin \theta d\theta = -2t dt$$

Hence, integral becomes,

$$I = \frac{1}{\sqrt{2k}} \int_1^{\sqrt{1/2}} \frac{-2t dt}{t}$$

$$= \sqrt{\frac{2}{k}} \int_1^{\frac{1}{\sqrt{2}}} dt$$

$$= \sqrt{\frac{2}{k}} \left(1 - \frac{1}{\sqrt{2}} \right)$$

$$= \frac{\sqrt{2} - 1}{\sqrt{k}}$$

$$= 1 - \frac{1}{\sqrt{2}} \text{ (Given)}$$

$$\therefore k = 2$$

94. (c) Let, $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$... (i)

$$\text{Using, } \int_a^b f(x) dx = \int_a^b f(a + b - x) dx, \text{ we get :}$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^{-x}} dx$$
 ... (ii)

Adding (i) and (ii), we get;

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x dx \Rightarrow 2I = 2 \cdot \int_0^{\pi/2} \sin^2 x dx$$

$$\Rightarrow 2I = 2 \times \frac{\pi}{4} \Rightarrow I = \frac{\pi}{4}$$

95. (a) $f(x) = \int_0^x t(\sin x - \sin t) \cdot dt$

$$= \sin x \int_0^x t \cdot dt - \int_0^x t \sin t \cdot dt$$

$$= \frac{x^2}{2} \sin x + [t \cos t]_0^x + \sin x$$

$$\Rightarrow f(x) = \frac{x^2}{2} \sin x + x \cos x + \sin x$$

$$f'(x) = \frac{x^2}{2} \cos x + 2 \cos x$$

$$f''(x) = x \cos x - \frac{x^2}{2} \sin x - 2 \sin x$$

$$f'''(x) = \cos x - 2x \sin x - \frac{x^2}{2} \cos x - 2 \cos x$$

$$\therefore f'''(x) + f'(x) = \cos x - 2x \sin x$$

96. (a) Let $I = \int_{\pi/4}^{3\pi/4} \frac{x}{1 + \sin x} dx$

$$\text{also let } K = \frac{x}{1 + \sin x}$$

Multiplying numerator and denominator by $(1 - \sin x)$, we get;

$$K = \frac{x(1 - \sin x)}{1 - (\sin x)^2} = \frac{x(1 - \sin x)}{(\cos x)^2}$$

$$= x(1 - \sin x) \sec^2 x$$

$$= x \sec^2 x - x \sin x \sec^2 x = x \sec^2 x - x \tan x \sec x$$

$$\text{Now, } I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x \sec^2 x dx - \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x \sec x \tan x dx$$

$$= \left[x \tan x - \int \frac{dx}{dx} \tan x dx \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} - \left[x \sec x - \int \frac{dx}{dx} \sec x dx \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \left[x \tan x - \ln |\sec x| \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$- \left[x \sec x - \ln |\sec x + \tan x| \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + c$$

$$\Rightarrow I = \left\{ \left[\frac{3\pi}{4} \tan \frac{3\pi}{4} - \ln \left| \frac{3\pi}{4} \right| \right] \right.$$

$$\left. - \left[\frac{3\pi}{4} \sec \frac{3\pi}{4} - \ln \left| \sec \frac{3\pi}{4} + \tan \frac{3\pi}{4} \right| \right] \right\}$$

$$- \left\{ \left[\frac{\pi}{4} \tan \frac{\pi}{4} - \ln \left| \frac{\pi}{4} \right| \right] \right.$$

$$\left. - \left[\frac{\pi}{4} \sec \frac{\pi}{4} - \ln \left| \sec \frac{\pi}{4} + \tan \frac{\pi}{4} \right| \right] \right\}$$

$$= \frac{\pi}{2} (\sqrt{2} + 1)$$

97. (d) Given:

$$I_1 = \int_0^1 e^{-x} \cos^2 x dx;$$

$$I_2 = \int_0^1 e^{-x^2} \cos^2 x dx \text{ and}$$

$$I_3 = \int_0^1 e^{-x^3} dx$$

For $x \in (0, 1)$

$$\Rightarrow x > x^2 \text{ or } -x < -x^2$$

$$\text{and } x^2 > x^3 \text{ or } -x^2 < -x^3$$

$$\therefore e^{-x^2} < e^{-x^3} \text{ and } e^{-x} < e^{-x^2}$$

$$\Rightarrow e^{-x} < e^{-x^2} < e^{-x^3}$$

$$\Rightarrow e^{-x^3} > e^{-x^2} > e^{-x}$$

$$\Rightarrow I_3 > I_2 > I_1$$

98. (c) Let

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \left(1 + \log \left(\frac{2 + \sin x}{2 - \sin x} \right) \right) dx \dots (1)$$

$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 (-x) \left(1 + \log \left(\frac{2 + \sin (-x)}{2 - \sin (-x)} \right) \right) dx$$

$$= \left[\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^4 x) \left(1 + \log \left(\frac{2 - \sin x}{2 + \sin x} \right) \right) dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x \left(1 - \log \left(\frac{2 + \sin x}{2 - \sin x} \right) \right) dx \dots (2)$$

After adding equation (1) and (2) we get,

$$2I = 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x dx$$

$$2I = 4 \int_0^{\frac{\pi}{2}} \sin^4 x dx$$

$$I = 2 \int_0^{\frac{\pi}{2}} \sin^4 x dx = \frac{2 \times \frac{3}{2} \times \frac{1}{2} \times \pi}{2 \times 2} = \frac{3\pi}{8}$$

[By Gamma function]

$$99. (c) I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x} \dots (i)$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 - \cos x} \dots (ii)$$

$$\text{Using } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx$$

Adding (i) and (ii)

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2}{\sin^2 x} dx; I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{cosec}^2 x dx$$

$$I = -(\cot x)_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = -\left[\cot \frac{3\pi}{4} - \cot \frac{\pi}{4} \right] = 2$$

$$100. (c) I_n = \int \tan^n x dx, n > 1$$

$$\text{Let } I = I_4 + I_6$$

$$= \int (\tan^4 x + \tan^6 x) dx = \int \tan^4 x \sec^2 x dx$$

Let $\tan x = t$

$$\Rightarrow \sec^2 x \, dx = dt$$

$$\therefore I = \int t^4 dt = \frac{t^5}{5} + C$$

$$= \frac{1}{5} \tan^5 x + C \Rightarrow \text{On comparing, we have}$$

$$a = \frac{1}{5}, b = 0$$

101. (a) Let $I = \int_1^2 \frac{dx}{((x-1a)^2 + 3)^{3/2}}$

Let; $x-1 = \sqrt{3} \tan \theta$

$$\Rightarrow dx = \sqrt{3} \sec^2 \theta \, d\theta$$

$$\Rightarrow I = \int_0^{\pi/6} \frac{\sqrt{3} \sec^2 \theta \, d\theta}{\left((\sqrt{3} \tan \theta)^2 + (\sqrt{3})^2 \right)^{3/2}}$$

$$= \frac{1}{3} \int_0^{\pi/6} \frac{\sec^2 \theta}{\sec^3 \theta} d\theta = \frac{1}{3} \int_0^{\pi/6} \cos \theta \, d\theta$$

$$= \frac{1}{3} [\sin \theta]_0^{\pi/6} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

$$= \frac{1}{6} = \frac{k}{k+5} \Rightarrow k+5 = 6k$$

$$\Rightarrow \boxed{k=1}$$

102. (a) $\int \frac{\frac{\pi}{4} \cos 2x}{\left(\frac{\pi}{12} \left(\frac{1}{\sin 2x} \right)^3 \right)} = \int \cos 2x \times \sin 2x \cdot \sin^2(2x) \, dx$

$$= \frac{1}{4} \int_{\pi/12}^{\pi/4} \sin 4x \cdot (1 - \cos 4x) \, dx$$

$$= \frac{1}{4} \left[\int_{\pi/12}^{\pi/4} \sin 4x - \frac{1}{2} \int_{\pi/12}^{\pi/4} \sin 8x \right]$$

$$= \frac{1}{4} \left[-\frac{\cos 4x}{4} + \frac{\cos 8x}{16} \right]_{\pi/12}^{\pi/4} = \frac{1}{4} \left[\frac{15}{32} \right] = \frac{15}{128}$$

103. (d) $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$

Dividing by x^{15} in numerator and denominator

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6} dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5} \right)^3}$$

$$\text{Let } 1 + \frac{1}{x^2} + \frac{1}{x^5} = t$$

$$\Rightarrow \left(\frac{-2}{x^3} - \frac{5}{x^6} \right) dx = dt \Rightarrow \left(\frac{2}{x^3} + \frac{5}{x^6} \right) dx = -dt$$

This gives,

$$\begin{aligned} \int \frac{\frac{2}{x^3} + \frac{5}{x^6} dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5} \right)^3} &= \int \frac{-dt}{t^3} = \frac{1}{2t^2} + C \\ &= \frac{1}{2 \left(1 + \frac{1}{x^2} + \frac{1}{x^5} \right)^2} + C = \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C \end{aligned}$$

104. (d) $x \int_1^x y(t) \, dt = x \int_1^x ty(t) \, dt + \int_1^x ty(t) \, dt$

Differentiate w.r. to x .

$$\int_1^x y(t) \, dt + x[y(x) - y(1)]$$

$$= \int_1^x ty(t) \, dt + x[xy(x) - y(1)] + xy(x) - y(1)$$

$$\int_1^x y(t) \, dt = \int_1^x ty(t) \, dt + x^2 y(x) - y(1)$$

Diff. again w.r. to x

$$y(x) - y(1) = xy(x) - y(1) + 2xy(x) + x^2 y'(x)$$

$$(1-3x)y(x) = x^2 y'(x)$$

$$\frac{y'(x)}{y(x)} = \frac{1-3x}{x^2}$$

$$\frac{1}{y} dy = \frac{1-3x}{x^2} \Rightarrow \ln y = -\frac{1}{x} - 3 \ln x$$

$$\ln(yx^3) = -\frac{1}{x}$$

$$yx^3 = -e^{-1/x}$$

$$y = \frac{e^{-1/x}}{x^3} \text{ or } y = \frac{ce^{-1/x}}{x^3}$$

105. (d) $I = \int_4^{10} \frac{[x^2]}{[x^2 - 28x + 196] + [x^2]} dx \dots (a)$

$$\text{Use } \int_a^b f(a+b-x) \, dx = \int_a^b f(x) \, dx$$

$$I = \int_4^{10} \frac{[(x-14)^2]}{[x^2] + [(x-14)^2]} dx \quad \dots(b)$$

(a) + (b)

$$2I = \int_4^{10} \frac{[(x-14)^2] + [x^2]}{[x^2] + [(x-14)^2]} dx$$

$$2I = \int_4^{10} dx \Rightarrow 2I = 6 \quad \Rightarrow I = 3$$

106. (b) $2 \int_0^1 \tan^{-1} x dx = \int_0^1 \left(\frac{\pi}{2} - \tan^{-1}(1-x+x^2) \right) dx$

$$2 \int_0^1 \tan^{-1} x dx = \int_0^1 \frac{\pi}{2} dx - \int_0^1 \tan^{-1}(1-x+x^2) dx$$

$$\int_0^1 \tan^{-1}(1-x+x^2) dx = \frac{\pi}{2} - 2 \int_0^1 \tan^{-1} x dx \quad \dots(a)$$

$$\text{Let, } I_1 = \int_0^1 \tan^{-1} x dx$$

$$= \left[(\tan^{-1} x)x \right]_0^1 - \int_0^1 \frac{1}{1+x^2} x dx$$

$$= \frac{\pi}{4} - \int_0^1 \frac{x}{1+x^2} dx = \frac{\pi}{4} - \frac{1}{2} \log 2$$

By equation (a)

$$\frac{\pi}{2} - 2 \left[\frac{\pi}{4} - \frac{1}{2} \log 2 \right] = \log 2$$

107. (a) $I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(36-12x+x^2)} dx$

$$I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(6-x)^2} dx \quad \dots(i)$$

$$I = \int_2^4 \frac{\log(6-x)^2}{\log(6-x)^2 + \log x^2} dx \quad \dots(ii)$$

Adding (i) and (ii)

$$2I = \int_2^4 dx = [x]_2^4 = 2$$

$$I = 1$$

108. (c) Let $f: R \rightarrow R$ be a function such that $f(2-x) = f(e+x)$

Put $x = 2+x$ we get

$$f(-x) = f(4+x) = f(4-x)$$

$$\Rightarrow f(x) = f(x+4)$$

Hence period is 4

$$\text{Consider } \int_{10}^{50} f(x) dx = 10 \int_{10}^{14} f(x) dx = 10 [5+5] = 100$$

109. (d) Let $f: (-1, 1) \rightarrow R$ be a continuous function

$$\text{Let } \int_0^{\sin x} f(t) dt = \frac{\sqrt{3}}{2} x$$

$$f(\sin x) \cdot \frac{d}{dx} (\sin x) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow f(\sin x) \cdot \cos x = \frac{\sqrt{3}}{2}$$

$$\text{put } x = \frac{\pi}{3}$$

$$f\left(\sin \frac{\pi}{3}\right) \cdot \cos \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$f\left(\frac{\sqrt{3}}{2}\right) \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$f\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

110. (c) $f\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{\ln t}{1+t} dt$

$$\text{Let } t = \frac{1}{z}$$

$$dt = -\frac{1}{z^2} dz$$

$$f(x) = \int_1^x \frac{\ln z}{z(z+1)} dz$$

$$f(x) + f\left(\frac{1}{x}\right) = \int_1^x \frac{\ln z}{z} dz = \left[\frac{(\ln z)^2}{2} \right]_1^x = \frac{(\ln x)^2}{2}$$

111. (b) Let $I = \int_0^{\pi} \sqrt{1+4\sin^2 \frac{x}{2}} - 4\sin \frac{x}{2} dx = \int_0^{\pi} \left| 2\sin \frac{x}{2} - 1 \right| dx$

$$= \int_0^{\pi/3} \left(1 - 2\sin \frac{x}{2} \right) dx + \int_{\pi/3}^{\pi} \left(2\sin \frac{x}{2} - 1 \right) dx$$

$$\left[\because \sin \frac{x}{2} = \frac{1}{2} \Rightarrow \frac{x}{2} = \frac{\pi}{6} \right]$$

$$\Rightarrow x = \frac{\pi}{3}, \frac{x}{2} = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{3} > \pi$$

$$\begin{aligned}
&= \left[x + 4 \cos \frac{x}{2} \right]_0^{\pi/3} + \left[-4 \cos \frac{x}{2} - x \right]_{\pi/3}^{\pi} \\
&= \frac{\pi}{3} + 4 \frac{\sqrt{3}}{2} - 4 + \left(0 - \pi + 4 \frac{\sqrt{3}}{2} + \frac{\pi}{3} \right) \\
&= 4\sqrt{3} - 4 - \frac{\pi}{3}
\end{aligned}$$

112. (d) $F(x) = \int_1^x \frac{e^t}{t} dt, x > 0$

Let $I = \int_1^x \frac{e^t}{t+a} dt$

Put $t + a = z \Rightarrow t = z - a; dt = dz$

for $t = 1, z = 1 + a$

for $t = x, z = x + a$

$$\therefore I = \int_{1+a}^{x+a} \frac{e^{z-a}}{z} dz$$

$$= e^{-a} \int_{1+a}^{x+a} \frac{e^z}{z} dz \equiv e^{-a} \int_{1+a}^{x+a} \frac{e^t}{t} dt$$

$$I = e^{-a} \left[\int_{1+a}^1 \frac{e^t}{t} dt + \int_1^{x+a} \frac{e^t}{t} dt \right]$$

$$= e^{-a} \left[-\int_1^{1+a} \frac{e^t}{t} dt + \int_1^{x+a} \frac{e^t}{t} dt \right]$$

$$= e^{-a} [-F(1+a) + F(x+a)]$$

(By the definition of $F(x)$)

$$= e^{-a} [F(x+a) - F(1+a)]$$

113. (a) Let $\int_{-\pi}^t (f(x) + x) dx = \pi^2 - t^2$

$$\Rightarrow \int_{-\pi}^t f(x) dx + \int_{-\pi}^t x dx = \pi^2 - t^2$$

$$\Rightarrow \int_{-\pi}^t f(x) dx + \left(\frac{t^2}{2} - \frac{\pi^2}{2} \right) = \pi^2 - t^2$$

$$\Rightarrow \int_{-\pi}^t f(x) dx = \frac{3}{2} (\pi^2 - t^2)$$

differentiating with respect to t

$$\frac{d}{dt} \left[\int_{-\pi}^t f(x) dx \right] = \frac{3}{2} \frac{d}{dt} (\pi^2 - t^2)$$

$$f(t) \cdot \frac{dt}{dt} - f(-\pi) \frac{d}{dt} (-\pi) = -3t$$

$$f(t) = -3t$$

$$f\left(-\frac{\pi}{3}\right) = -3\left(-\frac{\pi}{3}\right) = \pi$$

114. (d) Let $I = \int_0^{\pi} [\cos x] dx \dots (1)$

$$I = \int_0^{\pi} [\cos(\pi - x)] dx = \int_0^{\pi} [-\cos x] dx \dots (2)$$

On adding (1) and (2), we get

$$2I = \int_0^{\pi} [\cos x] dx + \int_0^{\pi} [-\cos x] dx$$

$$2I = \int_0^{\pi} [\cos x] + [-\cos x] dx$$

$$2I = \int_0^{\pi} -1 dx \quad (\because [x] + [-x] = -1 \text{ if } x \notin \mathbb{Z})$$

$$2I = -x \Big|_0^{\pi} = -\pi$$

$$\Rightarrow I = \frac{-\pi}{2}$$

115. (c) $P_n = \int_1^e (\log x)^n dx$

put $\log x = t$ then $x = e^t$ and $dx = e^t dt$

Also, when $x = 1$, then $t = \log 1 = 0$

and when $x = e$, then $t = \log_e e = 1$

$$\therefore P_n = \int_0^1 t^n \cdot e^t dt$$

$$\therefore P_{10} = \int_0^1 t^{10} e^t dt \text{ and } P_8 = \int_0^1 t^8 e^t dt$$

$$\text{Now, } P_{10} - 90P_8 = \int_0^1 t^{10} e^t dt - 90 \int_0^1 t^8 e^t dt$$

$$P_{10} - 90P_8 = \left[t^{10} e^t \right]_0^1 - 10 \int_0^1 t^9 e^t dt - 90 \int_0^1 t^8 e^t dt$$

$$P_{10} - 90P_8$$

$$= e - 10 \left[t^9 \int_0^1 e^t dt - \int_0^1 \frac{d}{dt} (t^9) \int_0^1 e^t dt \right] - 90 \int_0^1 t^8 e^t dt$$

$$P_{10} - 90P_8 = e - 10 \left[e - 9 \int_0^1 t^8 e^t dt \right] - 90 \int_0^1 t^8 e^t dt$$

$$P_{10} - 90P_8 = e - 10e + 90 \int_0^1 t^8 e^t dt - 90 \int_0^1 t^8 e^t dt$$

$$\therefore P_{10} - 90P_8 = -9e$$

116. (c) Let $I = \int_0^{1/2} \frac{\ln(1+2x)}{1+4x^2} dx$ or $\int_0^{1/2} \frac{\ln(1+2x)}{1+(2x)^2} dx$

Put $2x = \tan \theta$

$$\therefore \frac{2dx}{d\theta} = \sec^2 \theta \text{ or } dx = \frac{\sec^2 \theta d\theta}{2}$$

also when $x = 0 \Rightarrow \theta = 0$

and when $x = \frac{1}{2} \Rightarrow \theta = 45^\circ$ or $\frac{\pi}{4}$

$$\therefore I = \int_0^{\frac{\pi}{4}} \frac{\ln(1 + \tan \theta)}{1 + \tan^2 \theta} \times \frac{\sec^2 \theta d\theta}{2}$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{\ln(1 + \tan \theta)}{1 + \tan^2 \theta} \times \sec^2 \theta d\theta$$

($\because 1 + \tan^2 \theta = \sec^2 \theta$)

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \ln(1 + \tan \theta) d\theta \quad \dots(i)$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \ln \left[1 + \tan \left(\frac{\pi}{4} - \theta \right) \right] d\theta$$

(Using the property of definite integral)

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \ln \left[1 + \frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \times \tan \theta} \right] d\theta$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \ln \left[1 + \frac{1 - \tan \theta}{1 + \tan \theta} \right] d\theta$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \ln \left[\frac{1 + \cancel{\tan \theta} + 1 - \cancel{\tan \theta}}{1 + \tan \theta} \right] d\theta$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \ln \left[\frac{2}{1 + \tan \theta} \right] d\theta$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} [\ln 2 - \ln(1 + \tan \theta)] d\theta$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{4}} \ln 2 d\theta - \frac{1}{2} \int_0^{\frac{\pi}{4}} \ln(1 + \tan \theta) d\theta$$

$$I = \frac{1}{2} \ln 2 \theta \Big|_0^{\pi/4} - I \quad (\text{from eq. (i)})$$

$$I + I = \frac{1}{2} \ln 2 \left(\frac{\pi}{4} - 0 \right)$$

$$2I = \frac{1}{2} \times \frac{\pi}{4} \times \ln 2$$

$$2I = \frac{\pi}{8} \ln 2 \quad \text{or} \quad I = \frac{\pi}{16} \ln 2$$

$$117. (a) \text{ Since, } y = \int_0^x |t| dt, x \in \mathbb{R}$$

$$\text{therefore } \frac{dy}{dx} = |x|$$

$$\text{But from } y = 2x, \therefore \frac{dy}{dx} = 2$$

$$\Rightarrow |x| = 2 \Rightarrow x = \pm 2$$

$$\text{Points } y = \int_0^{\pm 2} |t| dt = \pm 2$$

$$\therefore \text{Equation of tangent is}$$

$$y - 2 = 2(x - 2) \text{ or } y + 2 = 2(x + 2)$$

$$\Rightarrow x\text{-intercept} = \pm 1.$$

$$118. (d) \text{ Let, } I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$$

$$= \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan \left(\frac{\pi}{2} - x \right)}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x} dx}{1 + \sqrt{\tan x}} \quad \dots(i)$$

Also, given

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x} dx}{1 + \sqrt{\tan x}} \quad \dots(ii)$$

By adding (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} dx$$

$$\Rightarrow I = \frac{1}{2} \left[\frac{\pi}{3} - \frac{\pi}{6} \right] = \frac{\pi}{12},$$

Statement-1 is false

$$\therefore \int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

It is fundamental property.

Statement -2 is true.

119. (a) Consider

$$\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t}) dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t}) dt$$

Let $I = f(x)$ after integrating and putting the limits.

$$f'(x) = \sin^{-1} \sqrt{\sin^2 x} (2 \sin x \cos x) - 0$$

$$+ \cos^{-1} \sqrt{\cos^2 x} (-2 \cos x \sin x) - 0$$

$$\therefore f'(x) = 0 \Rightarrow f(x) = C \quad (\text{constant})$$

Now, we find $f(x)$ at $x = \frac{\pi}{4}$

$$\therefore I = \int_0^{1/2} \sin^{-1} \sqrt{t} dt + \int_0^{1/2} \cos^{-1} \sqrt{t} dt$$

$$= \int_0^{1/2} (\sin^{-1} \sqrt{t} + \cos^{-1} \sqrt{t}) dt = \int_0^{1/2} \frac{\pi}{2} dt = \frac{\pi}{4} = C$$

$$\therefore f(x) = \frac{\pi}{4}$$

$$\therefore \text{Required integration} = \frac{\pi}{4}$$

$$120. (d) \quad I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx \quad \dots(i)$$

$$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}} dx, \text{ by replacing } x \text{ by } \left(\frac{\pi}{2} - \frac{\pi}{2} - x\right)$$

$$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{2^x \cdot \sin^2 x}{1+2^x} dx \quad \dots(ii)$$

Adding equations (i) and (ii), we get

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x dx = \frac{1}{2} \int_{-\pi/2}^{\pi/2} (1 - \cos 2x) dx$$

$$\Rightarrow I = \frac{1}{4} \left[x + \frac{\sin 2x}{2} \right]_{-\pi/2}^{\pi/2} \\ = \frac{1}{4} \left[\left(\frac{\pi}{2} + \frac{\sin \pi}{2} \right) - \left(-\frac{\pi}{2} + \frac{\sin(-\pi)}{2} \right) \right]$$

$$\Rightarrow I = \frac{1}{4} \left[\frac{\pi}{2} + \frac{\pi}{2} \right] = \frac{\pi}{4}$$

$$121. (d) \quad \text{Let } I = \int_{7\pi/4}^{7\pi/3} \sqrt{\tan^2 x} dx$$

$$= \int_{7\pi/4}^{7\pi/3} \tan x dx = -\log \cos x \Big|_{7\pi/4}^{7\pi/3}$$

$$= - \left[\log \cos \frac{7\pi}{3} - \log \cos \frac{7\pi}{4} \right]$$

$$= \log \cos \frac{7\pi}{4} - \log \cos \frac{7\pi}{3}$$

$$= \log \left[\frac{\cos \frac{7\pi}{4}}{\cos \frac{7\pi}{3}} \right] = \log \left[\frac{\cos \left(2\pi - \frac{\pi}{4} \right)}{\cos \left(2\pi + \frac{\pi}{3} \right)} \right]$$

$$= \log \left(\frac{\cos \frac{\pi}{4}}{\cos \frac{\pi}{3}} \right) = \log \left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} \right)$$

$$= \log \left(\frac{2}{\sqrt{2}} \right) = \log \sqrt{2}.$$

$$122. (a) \quad x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$$

$$\Rightarrow 1 = \frac{1}{\sqrt{1+y^2}} \cdot \frac{dy}{dx}$$

$$\left[\because \text{If } I(x) = \int_{\phi(x)}^{\psi(x)} f(t) dt, \text{ then } \frac{dI(x)}{dx} = f\{\psi(x)\} \cdot \left\{ \frac{d}{dx} \psi(x) \right\} - f\{\phi(x)\} \cdot \left\{ \frac{d}{dx} \phi(x) \right\} \right]$$

$$\left[\frac{d}{dx} \psi(x) \right] - f\{\phi(x)\} \cdot \left[\frac{d}{dx} \phi(x) \right]$$

$$\frac{dy}{dx} = \sqrt{1-y^2}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{1}{2\sqrt{1+y^2}} \cdot 2y \cdot \frac{dy}{dx} = \frac{y}{\sqrt{1+y^2}} \cdot \sqrt{1+y^2} = y$$

$$123. (b, c) \quad g(x+\pi) = \int_0^{x+\pi} \cos 4t dt$$

$$= \int_0^x \cos 4t dt + \int_x^{x+\pi} \cos 4t dt = g(x) + \int_0^\pi \cos 4t dt$$

(it is clear from graph of $\cos 4t$)

$$\int_x^{x+\pi} \cos 4t dt = \int_0^\pi \cos 4t dt = g(x) + g(\pi) = g(x) - g(\pi)$$

($\because$ From graph of $\cos 4t$, $g(\pi) = 0$)

$$124. (d) \quad \int_{-0.9}^{0.9} \left\{ [x^2] + \log \left(\frac{2-x}{2+x} \right) \right\} dx$$

$$= \int_{-0.9}^{0.9} [x^2] dx + \int_{-0.9}^{0.9} \log \left(\frac{2-x}{2+x} \right) dx$$

$$= 0 + \int_{-0.9}^{0.9} \log \left(\frac{2-x}{2+x} \right) dx$$

$$\text{Put } x = -x \Rightarrow f(x) = \log \frac{2-x}{2+x}$$

$$\text{and } f(-x) = \log \frac{2+x}{2-x} = -\log \frac{2-x}{2+x} = -f(x)$$

So, it is an odd function, hence

Required integral = 0.

$$125. (d) \quad \text{Since } \int_0^a [x] dx = 0 \text{ where } 0 \leq a \leq 1$$

$$\therefore \int_0^{0.9} [x - 2[x]] dx = 0$$

126. (a) Let $\frac{d}{dx} G(x) = \frac{e^{\tan x}}{x}, x \in \left(0, \frac{\pi}{2}\right)$

$$\text{Now, } I = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{2}{x} e^{\tan \pi x^2} \cdot dx = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{2\pi x}{\pi x^2} e^{\tan \pi x^2} \cdot dx$$

$$\text{Let } \pi x^2 = t \Rightarrow 2\pi x \, dx = dt$$

$$\text{When } x = \frac{1}{2}, t = \frac{\pi}{4} \text{ and } x = \frac{1}{4}, t = \frac{\pi}{16}$$

$$\therefore I = \int_{\frac{\pi}{16}}^{\frac{\pi}{4}} \frac{e^{\tan t}}{t} dt = G(t) \Big|_{\frac{\pi}{16}}^{\frac{\pi}{4}} = G\left(\frac{\pi}{4}\right) - G\left(\frac{\pi}{16}\right)$$

127. (d) Let $\int_e^x t f(t) dt = \sin x - x \cos x - \frac{x^2}{2}$

By using Leibnitz rule, we get

$$\frac{d}{dx} \left[\int_e^x t f(t) dt \right] = \frac{d}{dx} \left[\sin x - x \cos x - \frac{x^2}{2} \right]$$

$$\Rightarrow x f(x) - e f(e) = 0 = x \sin x - x$$

$$\text{Now, put } x = \frac{\pi}{6}, \text{ we get}$$

$$\frac{\pi}{6} \cdot f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} \cdot \sin \frac{\pi}{6} - \frac{\pi}{6}$$

$$\Rightarrow f\left(\frac{\pi}{6}\right) = \sin \frac{\pi}{6} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$$

128. (c) $\int_0^{1.5} x [x^2] dx = \int_0^1 x [x^2] dx + \int_1^{\sqrt{2}} x [x^2] dx + \int_{\sqrt{2}}^{1.5} x [x^2] dx$

$$= \int_0^1 x \cdot 0 \, dx + \int_1^{\sqrt{2}} x \, dx + \int_{\sqrt{2}}^{1.5} 2x \, dx = 0 + \left[\frac{x^2}{2} \right]_1^{\sqrt{2}} + \left[x^2 \right]_{\sqrt{2}}^{1.5}$$

$$= \frac{1}{2}(2-1) + (2.25-2) = \frac{1}{2} + 0.25$$

$$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

129. (d) $I = \int_0^1 \frac{8 \log(1+x)}{1+x^2} dx$

Put $x = \tan \theta$,

$$\therefore dx = \sec^2 \theta d\theta$$

$$\therefore I = 8 \int_0^{\pi/4} \frac{\log(1+\tan \theta)}{1+\tan^2 \theta} \cdot \sec^2 \theta d\theta$$

$$I = 8 \int_0^{\pi/4} \log(1+\tan \theta) d\theta \quad \dots(i)$$

$$\text{Applying } \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$= 8 \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - \theta \right) \right] d\theta$$

$$= 8 \int_0^{\pi/4} \log \left[1 + \frac{1-\tan \theta}{1+\tan \theta} \right] d\theta = 8 \int_0^{\pi/4} \log \left[\frac{2}{1+\tan \theta} \right] d\theta$$

$$= 8 \int_0^{\pi/4} [\log 2 - \log(1+\tan \theta)] d\theta$$

$$= 8 \log 2 \int_0^{\pi/4} 1 d\theta - 8 \int_0^{\pi/4} \log(1+\tan \theta) d\theta$$

$$I = 8 \cdot (\log 2) [x]_0^{\pi/4} - 8 \int_0^{\pi/4} \log(1+\tan \theta) d\theta$$

$$I = 8 \cdot \frac{\pi}{4} \cdot \log 2 - I$$

[From equation (i)]

$$\Rightarrow 2I = 2\pi \log 2$$

$$\therefore I = \pi \log 2$$

130. (a) $p'(x) = p'(1-x)$

$$\Rightarrow p(x) = -p(1-x) + c$$

at $x=0$

$$p(0) = -p(1) + c \Rightarrow 42 = c$$

$$\text{Now, } p(x) = -p(1-x) + 42$$

$$\Rightarrow p(x) + p(1-x) = 42$$

$$\text{Let } I = \int_0^1 p(x) dx \quad \dots(i)$$

$$\Rightarrow I = \int_0^1 p(1-x) dx \quad \dots(ii)$$

Adding eqn. (i) and (ii),

$$2I = \int_0^1 (42) dx \Rightarrow I = 21$$

131. (c) Let $I = \int_0^\pi [\cot x] dx \quad \dots(i)$

$$= \int_0^\pi [\cot(\pi-x)] dx = \int_0^\pi [-\cot x] dx \quad \dots(ii)$$

Adding eqn's (i) & (ii),

We get

$$2I = \int_0^\pi ([\cot x] + [-\cot x]) dx$$

$$= \int_0^\pi (-1) dx$$

$$[\because [x] + [-x] = -1, \text{ if } x \notin \mathbb{Z} \text{ and } [x] + [-x] = 0, \text{ if } x \in \mathbb{Z}]$$

$$= [-x]_0^\pi = -\pi \Rightarrow I = -\frac{\pi}{2}$$

132. (b) We know that $\frac{\sin x}{x} < 1$, for $x \in (0, 1)$

$$\Rightarrow \frac{\sin x}{\sqrt{x}} < \sqrt{x} \text{ on } x \in (0, 1)$$

$$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \sqrt{x} dx = \left[\frac{2x^{3/2}}{3} \right]_0^1$$

$$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \frac{2}{3} \Rightarrow I < \frac{2}{3}$$

$$\text{Also } \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}} \text{ for } x \in (0, 1)$$

$$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 x^{-1/2} dx = \left[2\sqrt{x} \right]_0^1 = 2$$

$$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}} dx < 2 \Rightarrow J < 2$$

133. (d) $\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$

$$\therefore \left[\sec^{-1} t \right]_{\sqrt{2}}^x = \frac{\pi}{2} \quad \left[\because \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x \right]$$

$$\Rightarrow \sec^{-1} x - \sec^{-1} \sqrt{2} = \frac{\pi}{2}$$

$$\Rightarrow \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \sec^{-1} x = \frac{\pi}{2} + \frac{\pi}{4}$$

$$\Rightarrow \sec^{-1} x = \frac{3\pi}{4} \Rightarrow x = \sec \frac{3\pi}{4} = \sec \left(\pi - \frac{\pi}{4} \right)$$

$$\Rightarrow x = -\sec \frac{\pi}{4} \Rightarrow x = -\sqrt{2}$$

134. (c) Given that $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where

$$f(x) = \int_1^x \frac{\log t}{1+t} dt$$

$$\therefore F(e) = f(e) + f\left(\frac{1}{e}\right)$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt \quad \dots (1)$$

$$\text{Let } I = \int_1^{1/e} \frac{\log t}{1+t} dt$$

$$\therefore \text{ Put } \frac{1}{t} = z \Rightarrow -\frac{1}{t^2} dt = dz \Rightarrow dt = -\frac{dz}{z^2}$$

$$\text{when } t = 1 \Rightarrow z = 1 \text{ and when } t = 1/e \Rightarrow z = e$$

$$\therefore I = \int_1^e \frac{\log \left(\frac{1}{z} \right)}{1 + \frac{1}{z}} \left(-\frac{dz}{z^2} \right)$$

$$= \int_1^e \frac{(\log 1 - \log z) \cdot z}{z+1} \left(-\frac{dz}{z^2} \right)$$

$$= \int_1^e -\frac{\log z}{(z+1)} \left(-\frac{dz}{z} \right) \quad [\because \log 1 = 0]$$

$$= \int_1^e \frac{\log z}{z(z+1)} dz$$

$$\therefore I = \int_1^e \frac{\log t}{t(t+1)} dt$$

$$[\text{By property } \int_a^b f(t) dt = \int_a^b f(x) dx]$$

Now from eqn. (1)

$$F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$$

$$= \int_1^e \frac{t \cdot \log t + \log t}{t(1+t)} dt = \int_1^e \frac{(\log t)(t+1)}{t(1+t)} dt$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{t} dt$$

$$\text{Let } \log t = x \quad \therefore \frac{1}{t} dt = dx$$

$$[\text{when } t = 1, x = 0 \text{ and when } t = e, x = \log e = 1]$$

$$\therefore F(e) = \int_0^1 x dx \quad F(e) = \left[\frac{x^2}{2} \right]_0^1$$

$$\Rightarrow F(e) = \frac{1}{2}$$

135. (b) Let $a = k + h$ where k is an integer such that and $0 \leq h < 1$

$$\Rightarrow [a] = k$$

$$\therefore \int_1^a [x] f'(x) dx = \int_1^2 1 f'(x) dx + \int_2^3 2 f'(x) dx +$$

$$\dots \int_{k-1}^k (k-1) f'(x) dx + \int_k^{k+h} k f'(x) dx$$

$$= \{f(2) - f(1)\} + 2\{f(3) - f(2)\} + 3\{f(4) - f(3)\} \\ + \dots + (k-1)\{f(k) - f(k-1)\} + k\{f(k+h) - f(k)\} \\ = -f(1) - f(2) - f(3) \dots - f(k) + kf(k+h) \\ = [a]f(a) - \{f(1) + f(2) + f(3) + \dots + f([a])\}$$

$$\text{136. (c)} \quad I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [(x+\pi)^3 + \cos^2(x+3\pi)] dx$$

$$\text{Put } x + \pi = t$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [t^3 + \cos^2 t] dt = 2 \int_0^{\frac{\pi}{2}} \cos^2 t dt$$

[$\because t^3$ is odd and $\cos^2 t$ is even function]

$$= \int_0^{\frac{\pi}{2}} (1 + \cos 2t) dt = \frac{\pi}{2} + 0$$

137. (d) $I = \int_0^{\pi} x f(\sin x) dx = \int_0^{\pi} (\pi - x) f(\sin x) dx$

$$= \pi \int_0^{\pi} f(\sin x) dx - I \Rightarrow 2I = \pi \int_0^{\pi} f(\sin x) dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx = \pi \int_0^{\pi/2} f(\sin x) dx$$

[$\because \sin(\pi - x) = \sin x$]

$$= \pi \int_0^{\pi/2} f(\cos x) dx$$

138. (b) $I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx \quad \dots(1)$

$$I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x}} dx \quad \dots(2)$$

$$[\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx]$$

Adding equation (1) and (2)

$$2I = \int_3^6 dx = [x]_3^6 = 3 \Rightarrow I = \frac{3}{2}$$

139. (b) Let $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx \quad \dots(1)$

$$= \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+a^{-x}} dx$$

$$\left[\text{Using } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

$$= \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1+a^x} dx \quad \dots(2)$$

Adding equations (1) and (2) we get

$$2I = \int_{-\pi}^{\pi} \cos^2 x \left(\frac{1+a^x}{1+a^x} \right) dx = \int_{-\pi}^{\pi} \cos^2 x dx$$

$$= 2 \int_0^{\pi} \cos^2 x dx \quad [\because f(\pi-x) = f(x)]$$

$$= 2 \times 2 \int_0^{\frac{\pi}{2}} \cos^2 x dx = 4 \int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \left[\because f\left(\frac{\pi}{2}-x\right) = f(x) \right]$$

$$\Rightarrow I = 2 \int_0^{\frac{\pi}{2}} \sin^2 x dx = 2 \int_0^{\frac{\pi}{2}} (1 - \cos^2 x) dx$$

$$\Rightarrow I = 2 \int_0^{\frac{\pi}{2}} dx - 2 \int_0^{\frac{\pi}{2}} \cos^2 x dx$$

$$\Rightarrow I + I = 2 \left(\frac{\pi}{2} \right) = \pi \Rightarrow I = \frac{\pi}{2}$$

140. (b) $I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx,$

$$I_3 = \int_0^1 2^{x^2} dx, I_4 = \int_0^1 2^{x^3} dx$$

$$\because 2^{x^3} < 2^{x^2}, 0 < x < 1$$

$$\Rightarrow \int_0^1 2^{x^2} dx > \int_0^1 2^{x^3} dx \Rightarrow I_1 > I_2$$

$$\text{and } 2^{x^3} > 2^x, x > 1$$

$$\Rightarrow I_4 > I_3$$

141. (d) $\lim_{x \rightarrow 2} \int_0^{f(x)} \frac{4t^3}{x-2} dt = \lim_{x \rightarrow 0} \frac{\int_0^{f(x)} 4t^3 dt}{x-2}$

Applying L Hospital rule

$$\lim_{x \rightarrow 2} \frac{[4f(x)^3 f'(x)]}{1} = 4(f(2))^3 f'(2)$$

$$= 4 \times 6^3 \times \frac{1}{48} = 18$$

142. (d) $f(x) = \frac{e^x}{1+e^x} \Rightarrow f(-x) = \frac{e^{-x}}{1+e^{-x}} = \frac{1}{e^x+1}$

$$\therefore f(x) + f(-x) = 1 \quad \forall x \in R$$

Now $I_1 = \int_{f(-a)}^{f(a)} x g\{x(1-x)\} dx$

$$= \int_{f(-a)}^{f(a)} (1-x) g\{x(1-x)\} dx$$

$$\left[\text{using } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

$$\Rightarrow \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx - \int_{f(-a)}^{f(a)} xg(x(1-x))dx$$

$$= I_2 - I_1 \Rightarrow 2I_1 = I_2$$

143. (b) Let $I = \int_0^{\pi} xf(\sin x)dx$... (i)

We know that

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx = \int_0^{\pi} (\pi-x)f(\sin x)dx \quad \dots (ii)$$

Adding (i) and (ii)

$$\therefore 2I = \pi \int_0^{\pi} f(\sin x)dx = \pi \cdot 2 \int_0^{\frac{\pi}{2}} f(\sin x)dx$$

$$[\because \sin(\pi-x) = \sin x]$$

$$\therefore I = \pi \int_0^{\frac{\pi}{2}} f(\sin x)dx \Rightarrow A = \pi$$

Let $\log x = t \Rightarrow e^t = x$

$$\Rightarrow \frac{1}{x}dx = dt \Rightarrow dx = xdt \Rightarrow e^t dt.$$

144. (c) $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$

$$\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}}$$

$$I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{(\sin x + \cos x)} dx = \int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx$$

$$\left[\because \sin x + \cos x > 0 \text{ if } 0 < x < \frac{\pi}{2} \right]$$

$$\text{or } I = [-\cos x + \sin x]_0^{\frac{\pi}{2}} = 2$$

145. (d) $\int_{-2}^3 |1-x^2| dx = \int_{-2}^3 |x^2-1| dx$

$$\text{Now } |x^2-1| = \begin{cases} x^2-1 & \text{if } x \leq -1 \\ 1-x^2 & \text{if } -1 \leq x \leq 1 \\ x^2-1 & \text{if } x \geq 1 \end{cases}$$

$$\therefore \text{Integral is } \int_{-2}^{-1} (x^2-1)dx + \int_{-1}^1 (1-x^2)dx + \int_1^3 (x^2-1)dx$$

$$= \left[\frac{x^3}{3} - x \right]_{-2}^{-1} + \left[x - \frac{x^3}{3} \right]_{-1}^1 + \left[\frac{x^3}{3} - x \right]_1^3$$

$$= \left(-\frac{1}{3} + 1 \right) - \left(-\frac{8}{3} + 2 \right) + \left(2 - \frac{2}{3} \right) + \left(\frac{27}{3} - 3 \right) - \left(\frac{1}{3} - 1 \right)$$

$$= \frac{2}{3} + \frac{2}{3} + \frac{4}{3} + 6 + \frac{2}{3} = \frac{28}{3}$$

146. (d) $I = \int_0^1 x(1-x)^n dx = \int_0^1 (1-x)(1-x)^n dx$

$$= \int_0^1 (1-x)x^n dx = \int_0^1 (x^n - x^{n+1}) dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} \right]_0^1 = \frac{1}{n+1} - \frac{1}{n+2}$$

147. (d) Given that $f'(x) = f(x) \Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrating both side we get

$$\log f(x) = x + c \Rightarrow f(x) = e^{x+c}$$

$$f(0) = 1 \Rightarrow f(x) = e^x$$

$$\therefore g(x) = x^2 - f(x) = x^2 - e^x$$

$$\therefore \int_0^1 f(x)g(x)dx = \int_0^1 e^x(x^2 - e^x)dx$$

$$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$$

$$= [x^2 e^x]_0^1 - 2[xe^x - e^x]_0^1 - \frac{1}{2}[e^{2x}]_0^1$$

$$= e - \left[\frac{e^2}{2} - \frac{1}{2} \right] - 2[e - e + 1] = e - \frac{e^2}{2} - \frac{3}{2}$$

148. (c) $I = \int_a^b xf(x)dx = \int_a^b (a+b-x)f(a+b-x)dx$

We know that

$$\therefore \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$= (a+b) \int_a^b f(a+b-x)dx - \int_a^b xf(a+b-x)dx$$

$$= (a+b) \int_a^b f(x)dx - \int_a^b xf(x)dx$$

[$\because$ Given that $f(a+b-x) = f(x)$]

$$2I = (a+b) \int_a^b f(x) dx$$

$$\Rightarrow I = \frac{(a+b)}{2} \int_a^b f(x) dx$$

$$149. (d) \lim_{x \rightarrow 0} \frac{\frac{d}{dx} \int_0^{x^2} \sec^2 t dt}{\frac{d}{dx} (x \sin x)} = \lim_{x \rightarrow 0} \frac{\sec^2 x^2 \cdot 2x}{\sin x + x \cos x}$$

(by L' Hospital rule)

$$\lim_{x \rightarrow 0} \frac{2 \sec^2 x^2}{\left(\frac{\sin x}{x} + \cos x\right)} = \frac{2 \times 1}{1+1} = 1$$

$$\begin{aligned} 150. (c) \quad F(t) &= \int_0^t f(t-y)g(y)dy \\ &= \int_0^t e^{t-y} y dy = e^t \int_0^t e^{-y} y dy \\ &= e^t \left[-ye^{-y} - e^{-y} \right]_0^t = -e^t \left[ye^{-y} + e^{-y} \right]_0^t \\ &= -e^t \left[te^{-t} + e^{-t} - 0 - 1 \right] = -e^t \left[\frac{t+1-e^t}{e^t} \right] \\ &= e^t - (1+t) \end{aligned}$$

$$\begin{aligned} 151. (b) \quad &\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx \\ &= \int_{-\pi}^{\pi} \frac{2x dx}{1+\cos^2 x} + 2 \int_{-\pi}^{\pi} \frac{x \sin x}{1+\cos^2 x} dx \\ &= 0 + 4 \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx \end{aligned}$$

We know that

$$\therefore \int_{-a}^a f(x) dx = 0, \text{ if } f(x) \text{ is odd.}$$

$$= 2 \int_0^a f(x) dx, \text{ if } f(x) \text{ is even}$$

$$I = 4 \int_0^{\pi} \frac{x(\pi-x) \sin(\pi-x)}{1+\cos^2(\pi-x)} dx$$

$$\begin{aligned} I &= 4 \int_0^{\pi} \frac{x(\pi-x) \sin x}{1+\cos^2 x} dx \\ \Rightarrow I &= 4\pi \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx - 4 \int_0^{\pi} \frac{x \sin x dx}{1+\cos^2 x} \end{aligned}$$

$$\Rightarrow 2I = 4\pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$$

$$\text{put } \cos x = t \Rightarrow -\sin x dx = dt$$

when $x=0$, $t=1$ and when $x=\pi$, $t=-1$

$$\begin{aligned} \therefore I &= -2\pi \int_1^{-1} \frac{1}{1+t^2} dt = 2\pi \int_{-1}^1 \frac{1}{1+t^2} dt \\ &= 2\pi \left[\tan^{-1} t \right]_{-1}^1 = 2\pi \left[\tan^{-1} 1 - \tan^{-1}(-1) \right] \\ &= 2\pi \left[\frac{\pi}{4} - \left(-\frac{\pi}{4} \right) \right] = 2\pi \cdot \frac{\pi}{2} = \pi^2 \end{aligned}$$

152. (d) We know that $[x]$ is greatest integer function less than equal to x

$$\begin{aligned} \therefore \int_0^2 [x^2] dx &= \int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2] dx + \\ &\quad \int_{\sqrt{2}}^{\sqrt{3}} [x^2] dx + \int_{\sqrt{3}}^2 [x^2] dx \end{aligned}$$

$$\begin{aligned} &= \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{\sqrt{3}} 2 dx + \int_{\sqrt{3}}^2 3 dx \\ &= [x]_0^1 + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^2 \\ &= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3} \\ &= 5 - \sqrt{3} - \sqrt{2} \end{aligned}$$

$$\begin{aligned} 153. (b) \quad I_n + I_{n+2} &= \int_0^{\pi/4} \tan^n x (1 + \tan^2 x) dx \\ &= \int_0^{\pi/4} \tan^n x \sec^2 x dx = \left[\frac{\tan^{n+1} x}{n+1} \right]_0^{\pi/4} \\ &\quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} \right] \\ &= \frac{1-0}{n+1} = \frac{1}{n+1} \end{aligned}$$

$$\therefore I_n + I_{n+2} = \frac{1}{n+1} \Rightarrow \lim_{n \rightarrow \infty} n [I_n + I_{n+2}]$$

$$= \lim_{n \rightarrow \infty} n \cdot \frac{1}{n+1} = \lim_{n \rightarrow \infty} \frac{n}{n+1} = \lim_{n \rightarrow \infty} \frac{n}{n \left(1 + \frac{1}{n} \right)} = 1$$

$$\begin{aligned} 154. (a) \quad I &= \int_0^{10\pi} |\sin x| dx = 10 \int_0^{\pi} |\sin x| dx \\ &\quad [\because \sin(10\pi - x) = \sin x] \end{aligned}$$

$$= 10 \int_0^{\pi} \sin x dx$$

 $\because \sin x > 0$, for $0 < x < \pi$.as $\sin(\pi - x) = \sin x$

$$I = 20 \int_0^{\pi/2} \sin x dx = 20 [-\cos x]_0^{\pi/2} = 20$$

$$155. (a) F(x) = \int_1^x t^2 g(t) dt$$

Differentiate by using Leibnitz's rule, we get

$$F'(x) = x^2 g(x) = x^2 \int_1^x f(u) du \quad \dots(i)$$

At $x=1$,

$$F'(1) = 1 \int_1^1 f(u) du = 0$$

Now, differentiate eqn (i)

$$F''(x) = x^2 f(x) - 2x \int_1^x f(u) du$$

At $x=1$,

$$F''(1) = 1 \cdot f(1) - 2 \times 1 \int_1^1 f(u) du$$

$$= f(1) - 2 \times 0 = f(1)$$

$$F''(1) = 3$$

Then, for $F'(1) = 0$, $F''(1) = 3 > 0$

Hence, $x=1$ is a point of local minima.

$$156. (a) \lim_{n \rightarrow \infty} \frac{(n+1)^{\frac{1}{3}} + (n+2)^{\frac{1}{3}} + \dots + (n+n)^{\frac{1}{3}}}{n(n)^{\frac{1}{3}}}$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{(n+r)^{\frac{1}{3}}}{n \cdot n^{\frac{1}{3}}}$$

$$= \int_0^1 (1+x)^{\frac{1}{3}} dx \quad \left[\because \frac{r}{n} \rightarrow x \text{ and } \frac{1}{n} \rightarrow \frac{dx}{x} \right]$$

$$= \left[\frac{3}{4} (1+x)^{\frac{4}{3}} \right]_0^1 = \frac{3}{4} (2)^{\frac{4}{3}} - \frac{3}{4}$$

$$157. (d) \text{ Let } L = \lim_{n \rightarrow \infty} \sum_{r=1}^{2n} \frac{n}{n^2 + r^2} = \int_0^2 \frac{dx}{1+x^2}$$

$$\left[\because \frac{r}{n} \rightarrow x, \frac{1}{n} \rightarrow \frac{dx}{r} \right]$$

$$= \left[\tan^{-1} x \right]_0^2$$

$$= \tan^{-1} 2$$

$$158. (a) \lim_{n \rightarrow \infty} \frac{\frac{1}{(a+1)} \cdot n^{a+1} + a_1 n^a + a_2 n^{a-1} + \dots}{(n+1)^{a-1} \cdot n^2 \left(a + \frac{1}{n} \right)} = \frac{1}{60}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{\left(\frac{1}{n} \right)^a + \left(\frac{2}{n} \right)^a + \dots + \left(\frac{n}{n} \right)^a}{(n+1)^{a-1} \left[n^2 a + \frac{n(n+1)}{2} \right]} = \frac{1}{60}$$

$$= \frac{\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n} \right)^a}{\left(1 + \frac{1}{n} \right)^{a-1} \left[a + \frac{1}{2} \left(1 + \frac{1}{n} \right) \right]} = \frac{1}{60}$$

$$= \frac{\int_0^1 x^a dx}{\left(a + \frac{1}{2} \right)} = \frac{1}{60} = \frac{\frac{1}{a+1}}{a + \frac{1}{2}} = \frac{1}{60}$$

$$\Rightarrow \frac{1}{\left(a + \frac{1}{2} \right)} = \frac{1}{60}$$

$$\Rightarrow (a+1)(2a+1) = 120$$

$$\Rightarrow 2a^2 + 3a - 119 = 0$$

$$\Rightarrow 2a^2 + 17a - 14a - 119 = 0$$

$$\Rightarrow (a-7)(2a+17) = 0$$

$$\Rightarrow a = 7, -\frac{17}{2}$$

$$159. (d) y = \lim_{n \rightarrow \infty} \left(\frac{(n+1)(n+2) \dots 3n}{n^{2n}} \right)^{\frac{1}{n}}$$

$$\ln y = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \left(1 + \frac{1}{n} \right) \left(1 + \frac{2}{n} \right) \dots \left(1 + \frac{2n}{n} \right)$$

$$\ln y = \lim_{n \rightarrow \infty} \frac{1}{n} \left[\ln \left(1 + \frac{1}{n} \right) + \ln \left(1 + \frac{2}{n} \right) + \dots + \ln \left(1 + \frac{2n}{n} \right) \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \ln \left(1 + \frac{r}{n} \right) = \int_0^2 \ln(1+x) dx$$

$$\text{Let } 1+x=t \Rightarrow dx=dt$$

$$\text{when } x=0, t=1$$

$$x=2, t=3$$

$$\ln y = \int_1^3 \ln t dt = [t \ln t - t]_1^3 = \ln \left(\frac{3^3}{e^2} \right) = \ln \left(\frac{27}{e^2} \right)$$

$$\Rightarrow y = \frac{27}{e^2}$$

160. (a) Let $f(x) = \int \frac{dx}{\sin^6 x}$

$$f(x) = \int \operatorname{cosec}^6 x dx$$

From reduction formula, we have

$$I_n = \int \operatorname{cosec}^n x dx = -\frac{\operatorname{cosec}^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} I_{n-2}$$

$$\therefore f(x) = -\frac{\operatorname{cosec}^4 x \cot x}{5} + \frac{4}{5} \left[-\frac{\operatorname{cosec}^2 x \cot x}{3} + \frac{2}{3} I_2 \right]$$

$$= -\frac{\operatorname{cosec}^4 x \cot x}{5} - \frac{4}{15} \operatorname{cosec}^2 x \cot x + \frac{8}{15} [-\cot x]$$

$$= \frac{-(1 + \cot^2 x)^2 \cot x}{5} - \frac{4}{15} (1 + \cot^2 x) \cot x - \frac{8}{15} (-\cot x) \quad (\because \operatorname{cosec}^2 x = 1 + \cot^2 x)$$

$$= \frac{-1}{5} [1 + \cot^4 x + 2 \cot^2 x] \cot x - \frac{4}{15} [\cot x + \cot^3 x] - \frac{8}{15} \cot x$$

$$= \frac{-1}{5} [\cot x + \cot^5 x + 2 \cot^3 x] - \frac{4}{15} \cot x - \frac{4}{15} \cot^3 x - \frac{8}{15} \cot x$$

$$= \frac{-15}{15} \cot x - \frac{\cot^5 x}{5} - \frac{10}{15} \cot^3 x$$

$$= \frac{-\cot^5 x}{5} - \frac{2}{3} \cot^3 x - \cot x$$

It is a polynomial of degree 5 in $\cot x$.

161. (d) $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \frac{3}{n^2} \sec^2 \frac{9}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right]$ is equal to

$$\lim_{n \rightarrow \infty} \frac{r}{n^2} \sec^2 \frac{r^2}{n^2} = \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \frac{r}{n} \sec^2 \frac{r^2}{n^2}$$

$\Rightarrow$ Given limit is equal to value of integral

$$\int_0^1 x \sec^2 x^2 dx$$

or $\frac{1}{2} \int_0^1 2x \sec^2 x^2 dx = \frac{1}{2} \int_0^1 \sec^2 t dt$ [put $x^2 = t$]

$$= \frac{1}{2} (\tan t)_0^1 = \frac{1}{2} \tan 1.$$

162. (b) $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} e^{\frac{r}{n}}$ [Using definite integrals as limit of sum]

$$= \int_0^1 e^x dx = e - 1$$

163. (a) $\lim_{n \rightarrow \infty} \frac{1^4 + 2^4 + 3^4 + \dots + n^4}{n^5}$

$$\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^5}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \left(\frac{r}{n} \right)^4 = \lim_{n \rightarrow \infty} \frac{1}{n} \cdot \lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{r}{n} \right)^3$$

$$= \int_0^1 x^4 dx = \lim_{n \rightarrow \infty} \frac{1}{n} \times \int_0^1 x^3 dx = \left[\frac{x^5}{5} \right]_0^1 = 0 = \frac{1}{5}$$

164. (a) We have $\lim_{n \rightarrow \infty} \frac{1^p + 2^p + \dots + n^p}{n^{p+1}}$;

$$\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{r^p}{n^p \cdot n} = \int_0^1 x^p dx = \left[\frac{x^{p+1}}{p+1} \right]_0^1 = \frac{1}{p+1}$$